
Homework 9

Exercise 1 *Differentiate the function* $\mathbb{R}_+ \ni x \mapsto \frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5} \in \mathbb{R}_+$.

Exercise 2 *Compute*
a) $\lim_{x\to 1}\left(\frac{x}{x-1} - \frac{1}{\ln(x)}\right)$,
b) $\lim_{x\to 0_+}\big(1+\sin(4x)\big)^{\cot(x)}$ *with* $\cot(x) = \frac{1}{\tan(x)}$.

Exercise 3 *Find the area under the following curves between the given bounds:*

1. $x \mapsto x^3$ *between* $x = 0$ *and* $x = 2$,
2. $x \mapsto \mathrm{e}^{-x}$ *between* $x = 0$ *and* $x = b > 0$, *what happens when* $b \to \infty$ *?*
3. $x \mapsto \cos(x) + \cos(2x)$ *between* $x = 0$ *and* $x = \pi/4$,
4. $x \mapsto x - \sin(x)$ *between* $x = 0$ *and* $x = \pi/2$,

and represent each of these areas on a drawing.

Exercise 4 *Let* $f : I \to \mathbb{R}$ *be a continuous function at* $b \in I$, *and let* g *be another function satisfying* $\lim_{x\to a} g(x) = b$. *Show that the following equality holds:*

$$\lim_{x\to a} f\big(g(x)\big) = f(b).$$