Nagoya University, G30 program Fall 2019

Mathematics Tutorial Ia (calculus) Instructor : Serge Richard

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Homework 5

Exercise 1 *Show that there are exactly two tangent lines to the graph of the function* $f : \mathbb{R} \ni x \mapsto (x+1)^2 \in \mathbb{R}$ *which pass through the origin. Find the equation of these lines (*$\Leftrightarrow$ *find the two functions whose graphs correspond to these straight lines).*

Exercise 2 *Consider the function* $f : \mathbb{R} \to \mathbb{R}$ *defined by* $f(x) = x^2 \sin(1/x)$ *if* $x \neq 0$ *and* $f(0) = 0$*.*

1. *Show that* f *is continuous at* 0*,*
2. *Compute the derivative of* f *at* 0*,*
3. *Compute the derivative of* f *at any* $x \neq 0$*,*
4. *Show that the derivative of* f *is well-defined but that this derivative is not continuous at* 0*.*

Indication: you can use that $\lim_{y\to 0} \frac{\sin(y)}{y} = 1$*.*

Exercise 3 *a) Let* $f(x) = x^2 \sin(1/x)$ *and* $g(x) = \sin(x)$ *for any* $x \in (-1,0) \cup (0,1)$*. Show that* $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$ *does not exist, but that* $\lim_{x\to 0} \frac{f(x)}{g(x)} = 0$*.*

b) Explain how this example fits in with L'Hospital's rule ?

Exercise 4 *Find the equation of the tangent of the curve in* $\mathbb{R}^2$ *defined by the relation*

$$F(x,y) = x^2 - y^2 + 3xy + 12 = 0$$

at the point $(-4, 2)$*.*

Exercise 5 *By using the indication mentioned above, compute the following limits:*

1. $\lim_{h\to 0} \frac{\cos(h)-1}{h}$ *,*
2. $\lim_{h\to 0} \frac{\cos(h)-1}{h^2}$ *.*