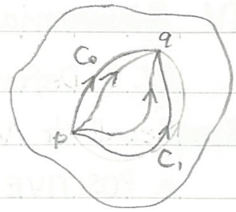

Consider a smooth map

$H: \underset{\nwarrow \text{parameter}}{[0,1]} \times [a,b] \mapsto M$

with $H(s,a) = p \in M$ and $H(s,b) = q \in M \quad \forall s \in [0,1]$

We set $\left.\begin{array}{l} C_0 : [a,b] \mapsto M,\ C_0(t) = H(0,t) \\ C_1 : [a,b] \mapsto M,\ C_1(t) = H(1,t) \end{array}\right\}$ We say that C_0 and C_1 are HOMOTOPIC paths between p and q.

Thm. Let $\omega \in \Lambda^1(M)$ s.t. $d\omega = 0$ everywhere. Then

$\int_{C_0} \omega = \int_{C_1} \omega$

Remark: if $\omega = d\phi$ with $\phi \in C^\infty(M) = \Lambda^0(M)$, then $d\omega = d^2\phi = 0$
and the statement follows from the previous lemma.

- If M is of dim 2, the statement is "almost" a consequence of Stoke's Thm, but we don't have the smoothness of the boundary at p and q.
- More generously, see [Bo p. 271]

Remark: Smoothness can be relaxed in most of the statements.