They are not so many holonomy groups!

Thm. Let (M, ϕ) and suppose that $Hol^{\circ}(M) \subset O(n)$ is irreducible $\overset{\text{no invariant}}{\text{subspaces of } \mathbb{R}^n}$.

(For a manifold made by product of two manifolds, this is not satisfied)

Suppose that M is not LOCALLY SYMMETRIC.

Then $Hol^{\circ}(M)$ is one of the following groups:

1) $SO(n)$ generic case
2) if $n=2m$, $Hol(M) = U(m)$
3) if $n=2m$, $Hol(M) = SU(m)$
4) $n=4m$, $Hol(M) = Sp(1)Sp(m)$
5) if $n=4m$, $Hol(M) = Sp(m)$
6) $n=16$, " $= Spin(9)$
7) $n=8$, " $= Spin(7)$
8) $n=7$, " $= G_2$

Later it is found that (6) does not actually appear in any manifolds.

4) ~ 8) are in qualernions $\overset{\text{extension of } \mathbb{C}}{2D \to 4D}$

Def. M is LOCALLY SYMMETRIC if for any $p \in M$:

the geodesic symmetry S_p is an isometry $\overset{\text{preserves the}}{\text{distance}}$

namely, we have $S_p(c(t)) = c(-t)$ for any geodesic c with $c(0) = p$

Example: $\mathbb{R}^n$ is locally symmetric (easy to show). And

$Hol(\mathbb{R}^n) = Hol^{\circ}(\mathbb{R}^n) = \{e\}$ which is not one of the 8 kinds of groups above.