
Homework 9

Exercise 1 Consider the function $\tanh : \mathbb{R} \rightarrow (-1, 1)$. Show that this function is invertible and compute the derivative of its inverse.

Exercise 2 Show that $\tanh(y)^{-1} = \frac{1}{2} \ln \left(\frac{1+y}{1-y} \right)$ for any $y \in (-1, 1)$.

Exercise 3 Determine the most general indefinite integral for the function $\mathbb{R}^* \ni x \mapsto \frac{1}{x} \in \mathbb{R}$.

Exercise 4 Find the area under the following curves between the given bounds:

1. $x \mapsto x^3$ between $x = 0$ and $x = 2$,
2. $x \mapsto e^{-x}$ between $x = 0$ and $x = b > 0$, what happens when $b \rightarrow \infty$?
3. $x \mapsto \cos(x) + \cos(2x)$ between $x = 0$ and $x = \pi/4$,
4. $x \mapsto x - \sin(x)$ between $x = 0$ and $x = \pi/2$,

and represent each of these areas on a drawing.

Exercise 5 For any $x \in \mathbb{R}$ with $x \neq -1$ we consider the sequence $(a_n)_{n \in \mathbb{N}}$ given by

$$a_n := \frac{x^n}{1 + x^n}.$$

For which x does the limit $\lim_{n \rightarrow \infty} a_n$ exists ? Give the value of this limit whenever it exists. Represent your findings on a graph (the horizontal axis corresponds to the x -variable).

Exercise 6 Let $f : I \rightarrow \mathbb{R}$ be a continuous function at $b \in I$, and let g be another function satisfying $\lim_{x \rightarrow a} g(x) = b$. Show that the following equality holds:

$$\lim_{x \rightarrow a} f(g(x)) = f(b).$$