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**Homework 2**

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**Exercise 1** Consider the sequences  $(a_n)_{n \in \mathbb{N}}$  defined below and show that these sequences are convergent. Can you find their limit ?

(i)  $a_n = \sqrt{n+1} - \sqrt{n}$ ,

(ii)  $a_n = \sqrt{n^2 + 5n} - n$ ,

(iii)  $a_n = \left(1 + \frac{1}{n}\right)^n$ .

In your proof you can use the equality

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k} = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$$

with  $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ .

**Exercise 2** Find the limit of the sequence  $\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots$

**Exercise 3** Show that the sequence  $(a_n)_{n \in \mathbb{N}}$  given by  $a_1 = 1$  and  $a_{n+1} = 3 - \frac{1}{a_n}$  for any  $n \geq 1$  is increasing and bounded from above by 3. Deduce that this sequence is convergent and give its limit.

**Exercise 4** Consider two real sequences  $(a_n)_{n \in \mathbb{N}}$  and  $(b_n)_{n \in \mathbb{N}}$  such that  $\lim_{n \rightarrow \infty} a_n = 0$  and  $|b_n| \leq C$  for one  $C > 0$  and all  $n \in \mathbb{N}$  (we say that the sequence  $(b_n)_{n \in \mathbb{N}}$  is bounded). Show that  $\lim_{n \rightarrow \infty} a_n b_n = 0$ .

A parametric curve on  $\mathbb{R}^2$  is a map

$$I \ni t \mapsto (x(t), y(t)) \in \mathbb{R}^2$$

where  $I$  is an interval of  $\mathbb{R}$ , and where  $x : I \rightarrow \mathbb{R}$  and  $y : I \rightarrow \mathbb{R}$  are real functions defined on  $I$ .

**Exercise 5** Represent the following parametric curves:

(i)  $x(t) = \cos(t)$  and  $y(t) = \sin(t)$  for any  $t \in [0, 2\pi]$ ,

(ii)  $x(t) = e^t \cos(t)$  and  $y(t) = e^t \sin(t)$  for any  $t \in \mathbb{R}$ ,

(iii)  $x(t) = \sin(t) - \sin(2.3t)$  and  $y(t) = \cos(t)$  for any  $t$ .