

Homework 13

A *parametric curve* on \mathbb{R}^2 is a map

$$I \ni t \mapsto (x(t), y(t)) \in \mathbb{R}^2$$

where I is an interval of \mathbb{R} , and where $x : I \rightarrow \mathbb{R}$ and $y : I \rightarrow \mathbb{R}$ are real functions defined on I .

Exercise 1 *Represent the following parametric curves:*

(i) $x(t) = \cos(t)$ and $y(t) = \sin(t)$ for any $t \in [0, 2\pi]$,

(ii) $x(t) = e^t \cos(t)$ and $y(t) = e^t \sin(t)$ for any $t \in \mathbb{R}$.

Exercise 2 *Consider the parametric curve defined by $x(t) = te^t$ and $y(t) = te^{-t}$ for any $t \in \mathbb{R}$. Determine the coordinates of the highest point on the curve, and of the leftmost point on the curve.*

Exercise 3 *Assume that the maps $I \ni t \mapsto x(t) \in \mathbb{R}$ and $I \ni t \mapsto y(t) \in \mathbb{R}$ are continuously differentiable (i.e. differentiable with a continuous derivative), then*

(i) *Determine the tangent line at any point of the parametric curve,*

(ii) *Determine the length of the parametric curve.*