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<u>Proposition</u>: If a translation bounded measure μ has an autocorrelation γ, then the measures $\{\gamma^L\}_{L=1}^{\infty}$ and γ are all translation bounded with constants independent of L.

<u>Proof</u>: $|\mu_L * \tilde{\mu}_L|(\chi_{K+x}) \leq (|\mu_L| * |\tilde{\mu}_L|)(\chi_{K+x})$ (property of convolution)

$$= \iint |\mu_L|(ds)\, |\tilde{\mu}_L|(dt)\, \chi_{K+x}(s+t)$$

$$= \iint |\mu_L|(ds)\, |\tilde{\mu}_L|(dt)\, \chi_{K+x-s}(t)$$

$$\leq \alpha_K \int |\mu_L|(ds) \quad \text{(translation boundedness)}$$

$$\leq \alpha_K\, \alpha_{C_1}\, L^d$$

number of cubes of side 1 needed to cover a cube of side L is L^d

$$\Rightarrow \gamma^L(K+x) = L^{-d}\, \mu_L * \tilde{\mu}_L(K+x) \leq 2\alpha_K\, \alpha_{C_1} \quad \forall L \geq 1, x$$

<u>Remark</u>: Every translation bounded measure has at least one autocorrelation.

<u>Lemma</u>: If a translation bounded measure μ has a unique autocorrelation, then $\gamma_\mu = \gamma_{\mu+\nu}$ $\forall$ bounded measure ν

<u>Proposition</u>: Let μ be a translation bounded measure that has a unique autocorrelation γ. Let $D \subset \mathbb{R}^d$ be s.t

$$\lim_{L\to\infty} L^{-d} \lambda(D_r \cap C_L) = 0 \quad \forall r > 0$$

where $D_r := \{x \in \mathbb{R}^d \mid \mathrm{dist}(x, D) < r\}$ and λ is the L. m.

Let ν be a translation bounded measure s.t $\nu(A) = 0$ if $A \cap D = \emptyset$ for all compact $A \subset \mathbb{R}^d$. Then $\gamma_\mu = \gamma_{\mu+\nu}$

<u>Proof</u>: $\lim_{L\to\infty} L^{-d}(\mu_L * \tilde{\mu}_L)(\phi) = \lim_{L\to\infty} L^{-d}(\mu_L + \nu_L) * (\tilde{\mu}_L + \tilde{\nu}_L)(\phi)$

$$\Rightarrow \lim_{L\to\infty} L^{-d} \mu_L * \tilde{\nu}_L(\phi) = 0 \text{ and } \lim_{L\to\infty} L^{-d} \nu_L * \tilde{\nu}_L(\phi) = 0$$

Consider: $\displaystyle\int |\tilde{\nu}_L|(t)(dt)\, \phi(s+t) = \int_{-D} |\tilde{\nu}_L|(t)\, \phi(s+t)\, dt + \underbrace{\int_{\mathbb{R}^d \setminus -D} |\tilde{\nu}_L|(t)\phi(s+t)\, dt}_{0 \text{ (by the assumption on } \nu)}$

$$= \int_{-D} |\tilde{\nu}|(t)\, \chi_{C_L}(t)\, \phi(s+t)\, dt.$$

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Denote A = support ϕ.

$\Rightarrow \phi(s+t) = 0$ if $s+t \notin A$

$\Rightarrow s + t \cap A = \emptyset$

$\Rightarrow s + (-D \cancel{\cap} C_L) \cap A = \emptyset$

$\Rightarrow (s - A) \cap D = \emptyset$

$\Rightarrow s \notin A + D$

$\Rightarrow \int |\tilde{\nu}_L|(t)\, \phi(s+t)\, dt = 0$ if $s \notin A+D$

By boundedness: $\int |\tilde{\nu}_L|(t)\, \phi(s+t)\, dt \leq C \chi_{A+D}(s)$

$$\Rightarrow |\mu_L * \tilde{\nu}_L|(\phi) \leq \int |\mu_L|(ds)\, |\tilde{\nu}_L|(dt)\, \phi(s+t)$$

$$\leq \int |\mu_L|(ds)\, C \chi_{A+D}(s)$$

$$= C \int \mu(s)\, \chi_{A+D\cap C_L}(s)\, ds$$

proof

$$\leq \alpha_{A+D\cap C_L}\, \lambda((D+A)\cap C_L)$$

Lebesgue measure

$$\mu(K+x) \leq \alpha_K \Rightarrow \mu(M) = \sum_{j=1}^{N} \mu(K_j) \leq N \alpha_K = \alpha_K \lambda(M)$$

unite cube — translation boundedness — N cubes K

$$\Rightarrow L^{-d} |\mu_L * \tilde{\nu}_L|(\phi) \leq \alpha_{A+D\cap C_L}\, L^{-d} \lambda((D+A)\cap C_L)$$

$$\Rightarrow \lim_{L\to\infty} \mu_L * \tilde{\nu}_L(\phi) = 0 \quad //$$