

Def. A directed set is a set A with

a reflexive ($\alpha \leq \alpha$) and transitive ($\beta \leq \gamma \Rightarrow \alpha \leq \gamma$) binary relation \leq such that $\forall \alpha, \beta \in A, \exists \gamma \in A : \alpha \leq \gamma, \beta \leq \gamma \ (\alpha, \beta \ can be uncomparable)$

Ex. · N, R

neighborhoods of $x \in X$, with $U \le V \Leftrightarrow U \supset V$

· {P| partation of [a,b]} $\ni P_1, P_2$, with $P_1 \le P_2 \Leftrightarrow P_2$ is finer than P_2

Def. A net in X is a map $A \ni A \mapsto X_A \in X$ with A a directed set (generalization of sequence)

Def: A net {Xa}aEA in X converges to x EX is if Y ngd. orhood U of x ∃do ∈A: ∀do ∈d, xd ∈U

·A point $x \in AX$ is a cluster point for a net $\{x_a\}_{a \in A}$ if YU ngd. of ix and for any XEA ∃BEA with X ≤ B and XBEU Ex. $0, 1, 0, 1, 0, 1, \cdots$ (0, 1 are cluster points)

Prop.

Let $E \subset X$, then $x \in E$ iff $\exists \not = a$ net $\{x_a\}_{\alpha \in A} \subset E$ converging to $\not= x$, and then XEE is an accumulation point of E is $\exists a \text{ net } \{x_a\}_{a \in A} \text{ converging } to \text{ in } E \setminus \{x\} \text{ to } x$

Remark: In metric spaces, a cluster point of a sequence

is the limit of a convergent subsequence. Similarly, a cluster point of a net is the limit of a convergent subnet.

Def. A subnet is a composition of A∋a → xa ∈ X with a new prop B∋B → dB ∈ X A with B another directed set such

Vdo ∈ A, ∃βo ∈ B with do ≤ dp for any βo ≤ β

Recall that in IRM, a set E is compact iff bounded & closed

Thm. If (X, p) is a metric space, and $E\subset X$, these are equivalent: • E is complete and totally bounded $(x \in X)$. E can be covered by a finite number of balls of radius E.)

- · Every sequence in E has a subsequence converging to a point in E

(*

· Every open cover of E has a finite subcover (For $X = \mathbb{R}^n$, they are equivalent to that E is bounded and closed)

In such a case we say that E is compact.

	Date · ·
Def. In (X, T), ECX is compact if (*):	Recall M(Rd) (all
every open cover of E admits a finite subcover.	$(g(x)\mu(ax))$
Remark: If $f: X \rightarrow Y$ with $(X, T)(Y, G) \ge top.$ space, is con-	tinuous 107
and if $E \subset X$ is compact, then $f(E)$ is compact in Y	Fall
Thm. Let (X, J) be a top. space. A set $E \subset X$ is compact.	iff () () ()
every net in E has a convergent subnet.	
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EM(K¹) for K>0 we sat	could that for us
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imit of this convergent subspect an autocorrelation for u	. We call the
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m for p. If line p. E extists in the vacque topology, we	o wer docorre o to

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