Nagoya University, G30 program Fall 2017

Mathematics Tutorial Ia (calculus) Instructor : Serge Richard

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Homework 10

Exercise 1 *Write out the lower and the upper Riemann sums for the function* $x \mapsto x^2$ *in the interval* $[0,2]$*. Use a regular partition of the interval divided into* n *subintervals of the same length. The following formula can be used:*

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

What happens when $n \to \infty$ *?*

Exercise 2 *Consider the function* $[0,1] \ni x \mapsto \mathrm{e}^x \in \mathbb{R}$*, and consider a regular partition of* $[0,1]$ *divided into* n *intervals of length* $\frac{1}{n}$*. Compute the following Riemann sums:*

1. $I_l := \sum_{j=0}^{n-1} \frac{1}{n} \mathrm{e}^{\frac{j}{n}}$ left rule,
2. $I_r := \sum_{j=1}^{n} \frac{1}{n} \mathrm{e}^{\frac{j}{n}}$ right rule,
3. $I_m := \sum_{j=0}^{n-1} \frac{1}{n} \mathrm{e}^{\frac{j+1/2}{n}}$ midpoint rule,
4. $I_{tri} := \frac{1}{2}\big(I_l + I_r\big)$ trapezoidal rule,
5. $I_{Sim} := \sum_{j=0}^{n/2-1} \frac{1}{3n}\big(\mathrm{e}^{\frac{2j}{n}} + 4\mathrm{e}^{\frac{2j+1}{n}} + \mathrm{e}^{\frac{2j+2}{n}}\big)$ *for* n *even* Simpson's rule.

Can you illustrate these rules on a drawing and compare their rate of convergence to their limit as $n \to \infty$ *? The following formula can be used for any* $a > 0$ *with* $a \neq 1$*:*

$$\sum_{k=0}^{m-1} a^k = \frac{a^m - 1}{a - 1}.$$

Exercise 3 (Mean value theorem for integrals) *Let* $f : [a,b] \to \mathbb{R}$ *be continuous. Show that there exists* $c \in (a,b)$ *such that*

$$\int_a^b f(x)\mathrm{d}x = f(c)\,(b-a).$$

Provide a geometric interpretation of this equality when f *is a positive function.*