Nagoya University, G30 program Fall 2016

Mathematics Tutorial Ia (calculus) Instructor : Serge Richard

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Homework 8

Exercise 1 *Prove the following statement:* Let $f : [a, b] \to \mathbb{R}$ be continuous on $[a, b]$ and differentiable on (a, b) with $f'(x) \geq 0$ for any $x \in (a, b)$. Then the function f is increasing on (a, b). Similarly, if $f'(x) \leq 0$ for any $x \in (a, b)$ then the function is decreasing on (a, b).

Exercise 2 *Determine the maximal domain on which the following functions are defined and sketch their graph as precisely as possible:*

$$a)\ f(x) = \frac{x-3}{x^2+1}, \qquad b)\ g(x) = \frac{2x^2-1}{x^2-2}, \qquad c)\ h(x) = x + \frac{1}{x}\ .$$

Exercise 3 *Prove the following statement:* Let $f : [a, b] \to \mathbb{R}$ be a strictly increasing and continuous function, and set $\alpha := f(a)$ and $\beta := f(b)$. Then there exists an inverse function $f^{-1} : [\alpha, \beta] \to [a, b]$ such that $f^{-1}\big(f(x)\big) = x$ and $f\big(f^{-1}(y)\big) = y$ for any $x \in [a, b]$ and $y \in [\alpha, \beta]$.

Exercise 4 *For the following functions f, determine whether there is an inverse function ? If not, determine some maximal domains for f on which local inverses can be defined.*

1. $f(x) = x^2 + 2x - 3$ *for* $x \leq 0$,
2. $f(x) = \frac{x}{x+1}$ *for* $-1 < x$,
3. $f(x) = x - \frac{1}{x}$ *for* $0 < x \leq 1$.

Exercise 5 *The function* hyperbolic tangent tanh *is defined for any $x \in \mathbb{R}$ by*

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

1. *Compute the derivative of* tanh *and find the critical point(s) of* tanh,
2. *Determine the maximal domain on which an inverse for this function can be defined, and call this inverse* arctanh,