
Homework 4

Exercise 1 Consider the curve in \mathbb{R}^2 defined by the relation

$$\frac{(x-1)^2}{9} + \frac{y^2}{4} = 1, \quad (x, y) \in \mathbb{R}^2.$$

Sketch this curve and determine the slope of the tangent at each point of it.

Exercise 2 For each integer n consider the polynomial function $p_n : \mathbb{R} \rightarrow \mathbb{R}$ defined by $p_n(x) = x^n$ and show that

$$p'_n(x) \equiv \frac{dp_n}{dx}(x) = nx^{n-1}.$$

Exercise 3 Consider the function f defined by $f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$ for any $x \in \mathbb{R}$.

1. For any fixed $x \in \mathbb{R}$ show that the sum is convergent,
2. Compute the derivative of f ,
3. What can you say about this function ?

Exercise 4 Compute the derivative of the functions $f : \mathbb{R} \rightarrow \mathbb{R}$ with $f(x)$ provided by the following expressions:

$$a) 5x^4 + 4x^2 - 1, \quad b) (x^5 + 1)(x^2 - 1), \quad c) \frac{5x-1}{x-5} \text{ for } x \neq 5, \quad d) \frac{x^{25} - 2x}{x^2 + 3}.$$

Exercise 5 Consider the function $f : \mathbb{R} \ni x \mapsto x^3 - 6x^2 + 8x \in \mathbb{R}$, and show that the graph of the function $\ell : \mathbb{R} \ni x \mapsto -x \in \mathbb{R}$ is tangent to the graph of the function f . Find the point of tangency.

Exercise 6 We say that a function $f : \mathbb{R} \rightarrow \mathbb{R}$ is uniformly continuous if for any $x \in \mathbb{R}$ and any $\varepsilon > 0$ there exists $\delta > 0$ (which depends on f and ε but NOT on x) such that $|f(x+h) - f(x)| \leq \varepsilon$ for any $|h| \leq \delta$. Show that the function defined by $f(x) = x^2$ is not uniformly continuous but that the function defined by $f(x) = x$ is uniformly continuous. What about the function f defined by $f(x) = \cos(x)$?