Nagoya University, G30 program — Fall 2016

Mathematics Tutorial Ia (calculus) — Instructor : Serge Richard

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Homework 2

Exercise 1 *Consider a sequence* $(a_n)_{n\in\mathbb{N}}$ *of real numbers (i.e.* $a_n \in \mathbb{R}$ *for any* $n \in \mathbb{N}$*). What is the precise meaning of* $\lim_{n\to\infty} a_n = a_\infty$ *? What could be the notion of* speed of convergence *? For example if* $a_n = 3 + (-1)^n \frac{1}{n}$ *or if* $a_n = 3 + \frac{2}{n^2}$*, what is the speed of convergence to* 3 *of the corresponding sequences ?*

Exercise 2 *Consider the sequences* $(a_n)_{n\in\mathbb{N}}$ *defined below and show that these sequences are convergent. Can you find their limit ?*

(i) $a_n = \sqrt{n+1} - \sqrt{n}$,

(ii) $a_n = \sqrt{n^2 + 5n} - n$,

(iii) $a_n = \left(1 + \frac{1}{n}\right)^n$.

In your proof you can use the equality

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

with $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Exercise 3 *Determine the function* $f : \mathbb{R} \to \mathbb{R}$ *whose graph is the straight line containing the points* (x_1, y_1) *and* (x_2, y_2) *of* $\mathbb{R}^2$*. What is the slope of this line ?*

A *parametric curve on* $\mathbb{R}^2$ is a map

$$I \ni t \mapsto \big(x(t), y(t)\big) \in \mathbb{R}^2$$

where I is an interval of $\mathbb{R}$, and where $x : I \to \mathbb{R}$ and $y : I \to \mathbb{R}$ are real functions defined on I.

Exercise 4 *Represent the following parametric curves:*

(i) $x(t) = \cos(t)$ *and* $y(t) = \sin(t)$ *for any* $t \in [0, 2\pi]$,

(ii) $x(t) = \mathrm{e}^t \cos(t)$ *and* $y(t) = \mathrm{e}^t \sin(t)$ *for any* $t \in \mathbb{R}$,

(iii) $x(t) = t + 2\sin(2t)$ *and* $y(t) = t + 2\cos(5t)$ *for any* $t \in [-4\pi, 4\pi]$.