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**Homework 1**

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**Exercise 1** Determine all numbers  $x \in \mathbb{R}$  satisfying the following inequalities:

$$a) |x - 1| < 2, \quad b) |2x - 3| \leq 4, \quad c) |x^2 - 1| \leq 1.$$

**Exercise 2** A function  $f : \mathbb{R} \rightarrow \mathbb{R}$  is said to be even if  $f(-x) = f(x)$  for any  $x \in \mathbb{R}$ , and  $f$  is said to be odd if  $f(-x) = -f(x)$  for any  $x \in \mathbb{R}$ .

1) Determine which of the functions defined for  $x \in \mathbb{R}$  by

$$a) f(x) = x, \quad b) f(x) = x^2, \quad c) f(x) = x^2 + x, \quad d) f(x) = \sin(x), \quad e) f(x) = \begin{cases} 1/x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

are even or odd ?

2) Show that for any function  $f$ , the function  $g : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $g(x) = \frac{1}{2}(f(x) + f(-x))$  is an even function while the function  $h : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $h(x) = \frac{1}{2}(f(x) - f(-x))$  is an odd function. In addition, observe that  $f = g + h$ .

3) What can you say about the graph of an even function, and about the graph of an odd function ?

**Exercise 3** Simplify the expression  $\left(\frac{5x^{3/2}y^4}{x^3y^{-1}}\right)^{-4}$ .

**Exercise 4** Consider the expression  $\frac{x^2+3x+2}{x^2+x-2}$ . Determine for which  $x \in \mathbb{R}$  this expression is well-defined, and simplify this expression.

**Exercise 5** If  $f : \mathbb{R} \ni x \mapsto x^2 \in \mathbb{R}$  and  $g : \mathbb{R}_+ \ni x \mapsto \sqrt{x} - 1 \in \mathbb{R}$ , determine the following functions if they are well-defined:

$$a) f \circ g, \quad b) f \circ f, \quad c) g \circ g.$$

**Exercise 6** Sketch the graph of the following function  $f : \mathbb{R} \rightarrow \mathbb{R}$  given for  $x \in \mathbb{R}$  by

$$a) f(x) = \frac{1}{2}x - 2, \quad b) f(x) = x - 3, \quad c) f(x) = x^2 + 3x + 2, \quad d) f(x) = \begin{cases} 1 + 1/(x - 2) & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}.$$

**Exercise 7** Consider a sequence  $(a_n)_{n \in \mathbb{N}}$  of real numbers (i.e.  $a_n \in \mathbb{R}$  for any  $n \in \mathbb{N}$ ). What is the precise meaning of  $\lim_{n \rightarrow \infty} a_n = 3$  ? What could be the notion of speed of convergence ? For example if  $a_n = 3 + (-1)^n \frac{1}{n}$  or if  $a_n = 3 + \frac{2}{n^2}$ , what is the speed of convergence to 3 of the corresponding sequences ?

**Exercise 8** Determine the equation of the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  whose graph is a straight line containing the points  $(x_1, y_1)$  and  $(x_2, y_2)$  of  $\mathbb{R}^2$ . What is the slope of this line ?