
Homework 8

Exercise 1 Consider $f : [a, b] \rightarrow \mathbb{R}$ continuous, and differentiable on (a, b) , and suppose that f' is also continuous on $[a, b]$. Show that the length ℓ of the curve defined by $\{(x, f(x)) \mid x \in [a, b]\}$ is given by the expression

$$\ell = \int_a^b \sqrt{1 + f'(x)^2} \, dx .$$

Exercise 2 Let $f : [a, b] \rightarrow \mathbb{R}_+$ be continuous and consider the volume of revolution generated by the rotation of $\{(x, f(x)) \mid x \in [a, b]\}$ around the x -axis. Show that the volume V of this solid is given by the expression

$$V = \pi \int_a^b f(x)^2 \, dx$$

Exercise 3 Let $f : [a, b] \rightarrow \mathbb{R}_+$ be continuous, and differentiable on (a, b) , and suppose that f' is also continuous on $[a, b]$. Consider the surface of revolution generated by the rotation of the points $\{(x, f(x)) \mid x \in [a, b]\}$ around the x -axis. Show that the surface S is given by the expression

$$S = 2\pi \int_a^b f(x) \sqrt{1 + f'(x)^2} \, dx .$$

Exercise 4 (The painter's paradox) Consider the function $f : [1, b] \ni x \mapsto \frac{1}{x} \in \mathbb{R}_+$ with $b > 1$. In the setting of the previous two exercises show that for any $b > 1$ one has $V = \pi(1 - \frac{1}{b})$ while $S > 2\pi \ln(b)$. By considering the limit $b \rightarrow \infty$ why do we get an apparent paradox ?