
Homework 6

Exercise 1 *Determine the maximal domain on which the following functions are defined, and compute their derivative:*

$$a)\ x \mapsto \arccos(2x), \qquad b)\ x \mapsto \arctan\big(1/x\big).$$

Sketch the graph of the second function.

Exercise 2 *Find the equation of the tangent at any point of the function* $\mathbb{R} \ni x \mapsto e^x \in \mathbb{R}_+$.

Exercise 3 *Recall that the hyperbolic cosine function is defined for* $x \in \mathbb{R}$ *by* $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$.

1. *Show that* $\cosh$ *is strictly increasing on* $\mathbb{R}_+^*$,
2. *Determine its inverse on* $\mathbb{R}_+$, *denoted by* $\operatorname{arccosh}$, *and compute the derivative of this function.*

Do the same exercise for the hyperbolic sine function $\sinh$ *on the entire line* $\mathbb{R}$.

Exercise 4 *Compute* $\ln(1)$ *and show that* $\ln(x^{-1}) = -\ln(x)$ *for any* $x \in \mathbb{R}_+^*$. *By setting*

$$a^{-x} := (a^x)^{-1}$$

for any $a, x \in \mathbb{R}_+^*$, *conclude that* $\ln(a^{-x}) = -x\ln(a)$.

Exercise 5 *For any* $r \in \mathbb{R}$, *consider the function* $\mathbb{R}_+ \ni x \mapsto f(x) := x^r \in \mathbb{R}_+$. *Compute the derivative of* f.

Exercise 6 *Determine the equation of the tangent of the curve* $y = \ln(x-2)$ *at the point* $x = 5$.

Exercise 7 *a) Let* $f(x) = x^2\sin(1/x)$ *and* $g(x) = \sin(x)$ *for any* $x \in (-1,0) \cup (0,1)$. *Show that* $\lim_{x\to 0}\frac{f'(x)}{g'(x)}$ *does not exist, but that* $\lim_{x\to 0}\frac{f(x)}{g(x)} = 0$.

b) Explain how this example fits in with L'Hospital's rule ?

Exercise 8 *Compute the following limits:*

$$a)\ \lim_{x\to 0_+} x\ln(x), \qquad b)\ \lim_{x\to 0_+} x^x, \qquad c)\ \lim_{x\to+\infty}\frac{\ln(x)}{x}, \qquad d)\ \lim_{x\to+\infty} x^{1/x}\ .$$

What can you say for $\lim_{x\to 0_+} x^r\ln(x)$ *for any* $r > 0$ *?*