Nagoya University, G30 program Fall 2015

Mathematics Tutorial Ia (calculus) Instructor : Serge Richard

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Homework 4

Exercise 1 *Provide two different examples of continuous functions on* $(-1,1)$ *which are not differentiable at* $x=0$.

Exercise 2 *Show that their are exactly two tangent lines to the graph of the function* $f:\mathbb{R}\ni x\mapsto (x+1)^2\in\mathbb{R}$ *which pass through the origin. Find the equation of these lines (*$\Leftrightarrow$ *find the two functions whose graphs correspond to these straight lines).*

Exercise 3 *1) For* $n\in\mathbb{N}^*$ *let* $P_{\frac{1}{n}}:\mathbb{R}_+\to\mathbb{R}$ *be the function defined by* $P_{\frac{1}{n}}(x)=x^{\frac{1}{n}}$. *Show that the following equality holds:*

$$P'_{\frac{1}{n}}(x)=\frac{1}{n}x^{\frac{1}{n}-1}$$

For the proof you can use the equality

$$(a^n-b^n)=(a-b)\sum_{k=0}^{n-1}a^{n-k-1}b^k$$

for $a=(x+h)^{1\frac{1}{n}}$ *and* $b=x^{\frac{1}{n}}$.

2) Deduce that if $P_{\frac{m}{n}}:\mathbb{R}_+\to\mathbb{R}$ *is defined for* $m,n\in\mathbb{N}^*$ *by* $P_{\frac{m}{n}}(x)=x^{\frac{m}{n}}$, *then*

$$P'_{\frac{m}{n}}(x)=\frac{m}{n}x^{\frac{m}{n}-1}\ .$$

Exercise 4 *Compute the derivative of the functions* $f:\mathbb{R}\to\mathbb{R}$ *defined for* $x\in\mathbb{R}$ *by*

$$a)\ \sin\big((2x+3)^2\big)\qquad b)\ \frac{(2x+3)^3}{(x-3)^2+1}\qquad c)\ \frac{1}{\sin(3x)^2+1}\qquad d)\ \mathrm{e}^{3\sin(x)+2}$$

Exercise 5 *Compute the derivatives of order* 1, 2 *and* 3 *for the functions* $f:\mathbb{R}\to\mathbb{R}$ *defined for* $x\in\mathbb{R}$ *by*

$$a)\ \cos(x)\qquad b)\ \cos(x)\sin(x)\qquad c)\ x^4+x^3+x^2+x^1+1$$

Exercise 6 *Find the equation of the tangent of the curve in* $\mathbb{R}^2$ *defined by the relation*

$$F(x,y)=x^2-y^2+3xy+12=0$$

at the point $(-4,2)$.