Nagoya University, G30 program Fall 2015

Mathematics Tutorial Ia (calculus) Instructor : Serge Richard

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Homework 3

Exercise 1 *Let* $f, g : \mathbb{R} \to \mathbb{R}$ *be two continuous functions. Show as precisely as possible that*

1. *the sum* $\lambda f + g$ *is continuous on* $\mathbb{R}$ *for any* $\lambda \in \mathbb{R}$*,*
2. *the product* fg *is continuous on* $\mathbb{R}$*,*

Exercise 2 *Let* I *be an open interval in* $\mathbb{R}$*, and let* $g : I \to \mathbb{R}$ *be a continuous function. If* $g(x) \neq 0$ *for some* $x \in I$*, show that there exists* $\delta > 0$ *such that* $g(x + h) \neq 0$ *for any* $h \in [-\delta, \delta]$*.*

Exercise 3 *Let* I *be an open interval in* $\mathbb{R}$*, and let* $f, g : I \to \mathbb{R}$ *be two differentiable functions at* $x \in I$*. If* $g(x) \neq 0$*, show that the following equality holds:*

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \ .$$

Exercise 4 *Compute the following limits, it they exist:*

1. $\lim_{x\to 0_+} \left(\frac{1}{x} - \frac{1}{|x|}\right)$ *and* $\lim_{x\to 0_-} \left(\frac{1}{x} - \frac{1}{|x|}\right)$,
2. $\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h}$,
3. $\lim_{x\to 2_+} \frac{x^2+x-6}{|x-2|}$ *and* $\lim_{x\to 2_-} \frac{x^2+x-6}{|x-2|}$,

Exercise 5 *Compute the derivative of the functions* $f : \mathbb{R} \to \mathbb{R}$ *with* $f(x)$ *provided by the following expressions:*

$$a)\ 7x^3 + 4x^2, \qquad b)\ (x^3 + x)(x - 1), \qquad c)\ \frac{2x-3}{x-5} \ \text{for } x \neq 5, \qquad d)\ \frac{x^2-2x}{x^2+3}.$$

Exercise 6 *Consider the function* $f : \mathbb{R} \ni x \mapsto x^3 - 6x^2 + 8x \in \mathbb{R}$*, and show that the graph of the function* $\ell : \mathbb{R} \ni x \mapsto -x \in \mathbb{R}$ *is tangent to the graph of the function* f*. Find the point of tangency.*