Nagoya University, G30 program Autumn 2014

Mathematics Tutorial I (linear algebra part) Instructor : Serge Richard

Quiz II

Name : ____________________

Exercise 1 *Let us consider*

$$\mathcal{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \qquad \textit{and} \qquad \mathcal{B} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

Compute $\mathcal{A}+\mathcal{B}$, $\mathcal{A}-2\mathcal{B}$, and ${}^t\mathcal{B}$.

$$\mathcal{A}+\mathcal{B} = \begin{pmatrix} 2 & 2 & 4 \\ 4 & 6 & 6 \end{pmatrix} \qquad \mathcal{A}-2\mathcal{B} = \begin{pmatrix} -1 & 2 & 1 \\ 4 & 3 & 6 \end{pmatrix}$$

$${}^t\mathcal{B} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Exercise 2 *Let us consider*

$$\mathcal{A} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \qquad \textit{and} \qquad \mathcal{B} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}.$$

Compute $\mathcal{A}\mathcal{B}$.

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 2 & 0 \\ 2 & 0 & 0 & 2 \\ 0 & 2 & 2 & 0 \\ 2 & 0 & 0 & 2 \end{pmatrix}$$

Exercise 3 *Let $\mathcal{A} = (a_{ij}) \in M_n(\mathbb{R})$. Write the conditions on a_{ij} such that $\mathcal{A}$ is*

1. *diagonal :* $a_{ij} = 0$ whenever $i \neq j$
2. *upper triangular :* $a_{ij} = 0$ whenever $i > j$
3. *symmetric :* $a_{ij} = a_{ji}$

Exercise 4 *What is the inverse of the matrix* $\begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$?

The inverse is given by $\begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}$.