

Name : _____

Exercise 1 For $A = (3, 2)$, $B = (-1, 1)$, $C = (0, -2)$, $\lambda = 3$, compute the following expressions :

$$i) A + B = (3, 2) + (-1, 1) = (2, 3).$$

$$ii) A - C = (3, 2) - (0, -2) = (3, 2) + (0, 2) = (3, 4).$$

$$iii) \lambda A + C = 3(3, 2) + (0, -2) = (9, 6) + (0, -2) = (9, 4).$$

$$iv) A \cdot B = (3, 2) \cdot (-1, 1) = -3 + 2 = -1.$$

$$v) \|A\| = \|(3, 2)\| = \sqrt{9+4} = \sqrt{13}.$$

$$vi) \text{the distance between } A \text{ and } C : d(A, C) = \|C - A\| = \|(-3, -4)\| = \sqrt{9+16} = 5.$$

Exercise 2 In the following cases, determine which located vectors \vec{PQ} and \vec{AB} are equivalent.

$$i) P = (-3, 2), Q = (0, 2), A = (1, -1), B = (4, 3)$$

$$\begin{aligned} Q - P &= (0, 2) - (-3, 2) = (0, 2) + (3, -2) = (3, 0) \\ B - A &= (4, 3) - (1, -1) = (4, 3) + (-1, 1) = (3, 4) \end{aligned}$$

$$ii) P = (-3, 0, 0), Q = (0, -4, 1), A = (1, -1, 5), B = (-2, 3, 4)$$

$$\begin{aligned} Q - P &= (0, -4, 1) - (-3, 0, 0) = (3, -4, 1) \\ B - A &= (-2, 3, 4) - (1, -1, 5) = (-3, 4, -1) \end{aligned}$$

Similarly, determine if these located vectors are parallel ?

Only in ii) they are parallel, since $(Q - P) = -1(B - A)$.

Exercise 3 Write a parametric representation of the line in \mathbb{R}^4 passing through the points $A = (1, 2, 3, 4)$ and $B = (1, 1, 1, 1)$.

$$\begin{aligned} L &= \{A + t(B - A) \mid t \in \mathbb{R}\} = \{(1, 2, 3, 4) + t(0, -1, -2, -3) \mid t \in \mathbb{R}\} \\ &= \{(1, 2-t, 3-2t, 4-3t) \mid t \in \mathbb{R}\}. \end{aligned}$$

Exercise 4 Find the equation of the hyperplane in \mathbb{R}^4 passing through the points $0 = (0, 0, 0, 0)$ and normal to $N = (1, 1, 1, 1)$.

$$H_{0, N} = \{x \in \mathbb{R}^4 \mid x \cdot N = 0\} = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_1 + x_2 + x_3 + x_4 = 0\}$$