Nagoya University, G30 program Autumn 2014

Linear algebra I Instructor : Serge Richard

Quiz II

Name : ______________________________

Exercise 1 *Write if the following statements are "true" or "false". Justify briefly your answer, or give a counterexample.*

1. *If $\mathcal{A}$ and $\mathcal{B}$ are symmetric matrices, then $\mathcal{A}+\mathcal{B}$ is symmetric,* True.

   If $a_{ij} = a_{ji}$ and $b_{ij} = b_{ji}$, then $(a_{ij} + b_{ij}) = (a_{ji} + b_{ji})$.

2. *If $\mathcal{A}$ is symmetric and $\mathcal{A} \neq \mathcal{O}$, then $\mathcal{A}$ is invertible,* False

   Counterexample : $\mathcal{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$.

3. *If $\mathcal{AB} = \mathcal{O}$, then either $\mathcal{A}$ or $\mathcal{B}$ is the matrix $\mathcal{O}$,* False

   Counterexample : $\mathcal{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $\mathcal{B} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$.

4. *If $\mathcal{A}^2 = \mathbf{1}$, then $\mathcal{A}$ is invertible,* True, with $\mathcal{A}^{-1} = \mathcal{A}$.

5. *If $\mathcal{A}, \mathcal{B}$ are invertible matrices, then $\mathcal{BA}$ is an invertible matrix,* True

   $(\mathcal{B}\mathcal{A})^{-1} = \mathcal{A}^{-1}\mathcal{B}^{-1}$.

6. *If $\mathcal{A} \in M_n(\mathbb{R})$, $B \in \mathbb{R}^n$ with $B \neq \mathbf{0}$, and if X and X' satisfy $\mathcal{A}X = B$ and $\mathcal{A}X' = B$, then $(X + X')$ satisfies the same equation,* False

   $\mathcal{A}(X+X') = \mathcal{A}X + \mathcal{A}X' = B + B = 2B \neq B$.

7. *If $\mathcal{A}$ is diagonal and if $\mathcal{B}$ is an arbitrary matrix, then the product $\mathcal{AB}$ is diagonal,* False

   Counterexample : $\mathcal{A} = \mathbb{1}_2$, $\mathcal{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\mathcal{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

8. *There exists an invertible matrix $\mathcal{A}$ such that $\mathcal{A}^{-1} = \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$,* False, since $\mathcal{A}^{-1}$ is not invertible (and one should have $(\mathcal{A}^{-1})^{-1} = \mathcal{A}$).

9. *Every matrices can be expressed as the product of elementary matrices,* False

   $\mathcal{O} \in M_n(\mathbb{R})$ is not such a product.

10. *$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is an orthogonal matrix.* True, because ${}^t\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

    and $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = {}^t\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.