Homework 3

Exercise 1

1) Recall that $X \in H_{P,N} \iff X \cdot N = P \cdot N$.

Then $X \cdot (\lambda N) = P \cdot (\lambda N) \iff \lambda (X \cdot N) = \lambda (P \cdot N)$

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$\iff X \cdot N = P \cdot N$ because $\lambda \neq 0$. It follows that $X \in H_{P,\lambda N} \iff X \in H_{P,N}$, and thus $H_{P,N} = H_{P,\lambda N}$.

2) Observe that $P' \in H_{P,N} \iff P' \cdot N = P \cdot N$. Then,

$X \in H_{P',N} \iff X \cdot N = P' \cdot N \iff X \cdot N = P \cdot N \iff X \in H_{P,N}$.

↑ by assumption $P' \in H_{P,N}$

Thus $X \in H_{P',N} \iff X \in H_{P,N}$, which means $H_{P,N} = H_{P',N}$.

Exercise 2

1) For the first plane, $N = (2, -1, 1)$, while for the second plane, $N' = (1, 2, -1)$. Thus $\cos \theta = \dfrac{N \cdot N'}{\|N\| \, \|N'\|} = \dfrac{-1}{\sqrt{6}\sqrt{6}} = -\dfrac{1}{6}$.

2) Similarly, $N = (1, 0, 0)$ and $N' = (3, 2, -7)$, and thus $\cos \theta = \dfrac{3}{1 \sqrt{62}} = \dfrac{3}{\sqrt{62}}$.

Exercise 3

One looks for $N = (n_1, n_2, n_3)$ such that $\overrightarrow{ON} \perp \overrightarrow{P_1P_2}$ and $\overrightarrow{ON} \perp \overrightarrow{P_1P_3}$. These condition are equivalent to $N \cdot (P_2 - P_1) = 0$ and $N \cdot (P_3 - P_1) = 0$, i.e. $-2n_1 - 1n_2 + 5n_3 = 0$ and $1n_2 - 1n_3 = 0$. Thus $n_2 = n_3$ and $-2n_1 = -4n_2$.

A possible solution is $N = (2, 1, 1)$ and

$$H_{P_1,N} = H_{P_2,N} = H_{P_3,N} = \{(x,y,z) \in \mathbb{R}^3 \mid 2x + y + z = 3\}.$$

Exercise 4

$$d(Q, H_{P,N}) \underset{\substack{\uparrow \\ \text{of course}}}{=} \frac{|(Q-P)\cdot N|}{\|N\|} = \frac{|(2,2,-2)\cdot(-1,1,-1)|}{\sqrt{3}} = \frac{2}{\sqrt{3}}.$$

Exercise 5

$L_{Q,N} = \{Q + tN \mid t \in \mathbb{R}\}$ is the line passing through Q and of direction N.

Then $X \in L_{Q,N} \cap H_{P,N}$ if and only if

$X = Q + tN$ for some $t \in \mathbb{R}$ and $X \cdot N = P \cdot N$

$\Leftrightarrow$ $X = Q + tN$ and $(Q + tN)\cdot N = P \cdot N$

$\Leftrightarrow$ $X = Q + tN$ and $tN^2 = P\cdot N - Q \cdot N$

$\Leftrightarrow$ $X = Q + tN$ and $t = \dfrac{(P-Q)\cdot N}{\|N\|^2} = \dfrac{(0,2,-1)\cdot(1,2,3)}{14} = \dfrac{1}{14}$.

Thus $X = Q + \frac{1}{14}N = \left(\frac{15}{14}, -\frac{6}{7}, \frac{31}{14}\right)$, which is the intersection of the line with the plane.

Exercise 6

1) The first plane is defined by $N = (1,2,3,4)$, and thus

$H_{P,N} = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid 1x_1 + 2x_2 + 3x_3 + 4x_4 = 1+2+3+4 = 10\}$.

2) More generally, if $N = (1,2,3,\dots,n)$ and $P = (1,1,1,\dots 1)$, then

$H_{P,N} = \{(x_1, x_2, \dots, x_n) \in \mathbb{R}^n \mid 1x_1 + 2x_2 + 3x_3 + \dots + nx_n = 1 + 2 + \dots + n = \frac{n(n+1)}{2}\}$.

3) Observe that $P = (1,1,\dots,1)$ does not belong to

$\{(x_1, x_2, \dots, x_n) \in \mathbb{R}^n \mid 1x_1 + 2x_2 + \dots + nx_n = n+1\}$ because

$n+1 \neq 1 + 2 + \dots + n = \frac{1}{2}n(n+1)$ except if $n = 2$.

Thus the 2 planes are parallel except for $n = 2$ in which case they are the same plane.