Exercise 1

	$A+B$	$A-B$	$3A$	$-2B$
1)	$(1,0)$	$(3,-2)$	$(6,-3)$	$(2,-2)$
2)	$(1,0,6)$	$(3,-2,4)$	$(6,-3,15)$	$(2,-2,-2)$
3)	$(3\pi,0,6)$	$(-\pi,6,-8)$	$(3\pi,9,-3)$	$(-4\pi,6,-14)$

Exercise 2

$A+2B=(7,4)$ $\quad$ $A-3B=(-8,-1)$ $\quad$ $A+\frac{1}{2}B=\left(\frac{5}{2},\frac{5}{2}\right)$

Exercise 3

Let $A=(a_1,\dots,a_n)$, $B=(b_1,\dots,b_n)$, $C=(c_1,\dots,c_n)$

1) $A+B=(a_1+b_1,\dots,a_n+b_n)=(b_1+a_1,\dots,b_n+a_n)=B+A$.

2) $(A+B)+C=(a_1+b_1,\dots,a_n+b_n)+(c_1,\dots,c_n)$
$$=(a_1+b_1+c_1,\dots,a_n+b_n+c_n)$$
$$=(a_1,\dots,a_n)+(b_1+c_1,\dots,b_n+c_n)$$
$$=A+(B+C).$$

3) $\lambda(A+B)=\lambda(a_1+b_1,\dots,a_n+b_n)=(\lambda(a_1+b_1),\dots,\lambda(a_n+b_n))$
$$=(\lambda a_1+\lambda b_1,\dots,\lambda a_n+\lambda b_n$$
$$=(\lambda a_1,\dots,\lambda a_n)+(\lambda b_1,\dots,\lambda b_n)$$
$$=\lambda A+\lambda B.$$

4) $(\lambda+\mu)A=((\lambda+\mu)a_1,\dots,(\lambda+\mu)a_n)=(\lambda a_1+\mu a_1,\dots,\lambda a_n+\mu a_n)$
$$=(\lambda a_1,\dots,\lambda a_n)+(\mu a_1,\dots,\mu a_n)$$
$$=\lambda A+\mu A.$$

5) $(\lambda\mu)A = ((\lambda\mu)a_1, \dots, (\lambda\mu)a_n)$
$= (\lambda\mu a_1, \dots, \lambda\mu a_n)$
$= \lambda(\mu a_1, \dots, \mu a_n)$
$= \lambda(\mu A)$.

Exercise 4

1) not equivalent
2) equivalent
3) not equivalent
4) equivalent

1) $Q-P=(3,4)$, $B-A=(8,-4)$
no λ such that $(3,4)=\lambda(8,-4)$
$\Rightarrow \vec{AB}$ not parallel to $\vec{PQ}$.

2) $Q-P=(-4,1)$, $B-A=(4,-1)$
$\Rightarrow \vec{AB}$ parallel to $\vec{PQ}$, with $\lambda=-1$.

3) $Q-P=(-3,4,-9)$, $B-A=(-6,8,-18) \Rightarrow \vec{AB}$ parallel to $\vec{PQ}$
since $(-3,4,-9)=\frac{1}{2}(-6,8,-18)$.

4) $Q-P=(-3,0,9)$, $B-A=(-9,0,-27)$, $\vec{AB}$ not parallel to $\vec{PQ}$.

Exercise 5

1) $A\cdot A=5$ and $A\cdot B=-3$
2) $A\cdot A=30$ and $A\cdot B=2$
3) $A\cdot A=10+\pi^2$ and $A\cdot B=2\pi^2-16$
4) $A\cdot A=3$ and $A\cdot B=0$.

Only the pair 4) contains perpendicular vectors.

Exercise 6

1) $A \cdot B = \sum_{j=1}^{n} a_j b_j = \sum_{j=1}^{n} b_j a_j = B \cdot A.$

2) $A \cdot (B+C) = \sum_{j=1}^{n} a_j (b_j + c_j) = \sum_{j=1}^{n} (a_j b_j + a_j c_j)$

$$= \sum_{j=1}^{n} a_j b_j + \sum_{j=1}^{n} a_j c_j = A \cdot B + A \cdot C.$$

3) $(\lambda A) \cdot B = \sum_{j=1}^{n} (\lambda a_j) b_j = \sum_{j=1}^{n} \lambda a_j b_j$

$$= \lambda \sum_{j=1}^{n} a_j b_j = \lambda (A \cdot B).$$

4) $A \cdot A = \sum_{j=1}^{n} a_j a_j = \sum_{j=1}^{n} a_j^2 \geqslant 0$ with an equality if and only if $a_j^2 = 0 \ \forall j = 1, \dots, n$, i.e. if and only if $A = 0$.

Exercise 7

1) $(A+B)^2 = (A+B) \cdot (A+B) = (A+B) \cdot A + (A+B) \cdot B$

$$= A^2 + B \cdot A + A \cdot B + B^2 = A^2 + 2 A \cdot B + B^2.$$

2) $(A-B)^2 = (A-B) \cdot (A-B) = (A-B) \cdot A + (A+B) \cdot (-B)$

$$= A^2 - B \cdot A - A \cdot B + B^2.$$