Nagoya University, G30 program Autumn 2014

Mathematics Tutorial I (linear algebra part) Instructor : Serge Richard

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Homework 5

Exercise 1 *Let us consider*

$$\mathcal{A} = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{pmatrix} \qquad \text{and} \qquad \mathcal{B} = \begin{pmatrix} -1 & 5 & -2 \\ 1 & 1 & -1 \end{pmatrix}.$$

Compute $\mathcal{A}+\mathcal{B}$, $\mathcal{A}-2\mathcal{B}$, *and* ${}^t\mathcal{A}$.

Exercise 2 *(i) For any* $\mathcal{A} \in M_{mn}(\mathbb{R})$, $\mathcal{B}, \mathcal{C} \in M_{np}(\mathbb{R})$ *and* $\lambda \in \mathbb{R}$ *show that*

(a) $\mathcal{A}(\mathcal{B}+\mathcal{C}) = \mathcal{A}\mathcal{B}+\mathcal{A}\mathcal{C}$,

(b) $(\lambda\mathcal{A})\mathcal{B} = \lambda(\mathcal{A}\mathcal{B}) = \mathcal{A}(\lambda\mathcal{B})$.

(ii) If $\mathcal{A} \in M_{mn}(\mathbb{R})$, $\mathcal{B} \in M_{np}(\mathbb{R})$ *and* $\mathcal{C} \in M_{pq}(\mathbb{R})$ *show that*

$$(\mathcal{A}\mathcal{B})\mathcal{C} = \mathcal{A}(\mathcal{B}\mathcal{C}).$$

(iii) If $\mathcal{A} \in M_{mn}(\mathbb{R})$ *and* $\mathcal{B} \in M_{np}(\mathbb{R})$ *show that*

$${}^t(\mathcal{A}\mathcal{B}) = {}^t\mathcal{B}\;{}^t\mathcal{A}.$$

Exercise 3 *Let* $\mathcal{A} \in M_{mn}(\mathbb{R})$. *Show that* $\mathbf{1}_m\,\mathcal{A} = \mathcal{A} = \mathcal{A}\mathbf{1}_n$.

Exercise 4 *One says that a matrix* $\mathcal{A} \in M_n(\mathbb{R})$ *is* symmetric *if* ${}^t\mathcal{A} = \mathcal{A}$ *and is* skew-symmetric *if* ${}^t\mathcal{A} = -\mathcal{A}$. *Show that for an arbitrary matrix* $\mathcal{A} \in M_n(\mathbb{R})$, *the matrix* $\mathcal{A} + {}^t\mathcal{A}$ *is symmetric while the matrix* $A - {}^t\mathcal{A}$ *is skew-symmetric.*

Exercise 5 *Let* $\mathcal{A} \in M_n(\mathbb{R})$.

1. *If* $\mathcal{A}^2 = \mathcal{O}$, *show that* $\mathbf{1}_n - \mathcal{A}$ *is invertible.*
2. *Suppose that* $\mathcal{A}^2 + 2\mathcal{A} + \mathbf{1}_n = \mathcal{O}$. *Show that* $\mathcal{A}$ *is invertible.*

Exercise 6

1. *Find some* $\mathcal{A} \in M_2(\mathbb{R})$ *such that* $\mathcal{A}^2 = -\mathbf{1}_2$.
2. *Determine all* $\mathcal{A} \in M_2(\mathbb{R})$ *such that* $\mathcal{A}^2 = \mathcal{O}$.