
Homework 4

Exercise 1 Let $X = (x_1, x_2, x_3)$ and $Y = (y_1, y_2, y_3)$ be vectors in \mathbb{R}^3 , and recall that $X \times Y$ is given by

$$X \times Y = \begin{pmatrix} x_2 y_3 - x_3 y_2 \\ x_3 y_1 - x_1 y_3 \\ x_1 y_2 - x_2 y_1 \end{pmatrix}.$$

- (i) Show that $(X + X') \times Y = X \times Y + X' \times Y$ and $X \times Y = -Y \times X$,
- (ii) Show that $X \times Y$ is perpendicular to X and to Y ,
- (iii) Show the following relation:

$$\|X \times Y\|^2 = \|X\|^2 \|Y\|^2 - (X \cdot Y)^2,$$

- (iv) Deduce that $\|X \times Y\| = \|X\| \|Y\| |\sin(\theta)|$, where θ is the angle between the vectors X and Y ,
- (v) More generally, if $X, Y \in \mathbb{R}^n$ show the following relation:

$$\sum_{1 \leq i < j \leq n} (x_i y_j - x_j y_i)^2 = \|X\|^2 \|Y\|^2 - (X \cdot Y)^2.$$