
Homework 3

Exercise 1 For any $P, N \in \mathbb{R}^n$ with $N \neq \mathbf{0}$, and for any $\lambda \in \mathbb{R}^*$ show that

(i) $H_{P,N} = H_{P,\lambda N}$,

(ii) If $P' \in H_{P,N}$, then $H_{P',N} = H_{P,N}$.

Exercise 2 Determine the cosine of the angle between the two planes defined by

$$\{(x, y, z) \in \mathbb{R}^3 \mid 2x - y + z = 0\} \quad \text{and} \quad \{(x, y, z) \in \mathbb{R}^3 \mid x + 2y - z = 1\}.$$

Same question for the planes defined by

$$\{(x, y, z) \in \mathbb{R}^3 \mid x = 1\} \quad \text{and} \quad \{(x, y, z) \in \mathbb{R}^3 \mid 3x + 2y - 7z = 1\}.$$

Exercise 3 Find the equation of the plane in \mathbb{R}^3 passing through the three points $P_1 = (1, 2, -1)$, $P_2 = (-1, 1, 4)$ and $P_3 = (1, 3, -2)$.

Exercise 4 Let $P = (-1, 1, 7)$, $Q = (1, 3, 5)$ and $N = (-1, 1, -1)$. Determine the distance between the point Q and the plane $H_{P,N}$.

Exercise 5 Let $P = (1, 1, 1)$, $Q = (1, -1, 2)$ and $N = (1, 2, 3)$. Find the intersection of the line passing through Q and having the direction N with the plane $H_{P,N}$.

Exercise 6 Determine the equation of the hyperplane in \mathbb{R}^4 passing through the point $(1, 1, 1, 1)$ and which is parallel to the hyperplane defined by

$$\{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid 1x_1 + 2x_2 + 3x_3 + 4x_4 = 5\}.$$

Similarly, for any $n > 1$ determine the equation of the hyperplane in \mathbb{R}^n passing through the point $(1, 1, \dots, 1)$ and which is parallel to the hyperplane defined by

$$\{(x_1, x_2, \dots, x_n) \in \mathbb{R}^n \mid \sum_{j=1}^n j x_j = n + 1\}.$$

Does something special happen for $n = 2$?