Nagoya University, G30 program Autumn 2014

Mathematics Tutorial I (linear algebra part) Instructor : Serge Richard

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Homework 11

Exercise 1 *Show that the following sets of elements of* $\mathbb{R}^3$ *form subspaces :*

i) $S_1 := \{{}^t(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0\}$,

ii) $S_2 := \{{}^t(x, y, z) \in \mathbb{R}^3 \mid x = y \text{ and } 2y = z\}$,

iii) $S_3 := \{{}^t(x, y, z) \in \mathbb{R}^3 \mid x + y = 3z\}$.

Exercise 2 *Let* V *be a subspace of* $\mathbb{R}^n$*, and let* W *be the set of all elements of* $\mathbb{R}^n$ *which are perpendicular to all elements of* V*. Show that* W *itself is a subspace of* $\mathbb{R}^n$*. This subspace is often denoted by* $V^\perp$ *and called the orthogonal complement of* V *in* $\mathbb{R}^n$*.*

Exercise 3 *Let* $X_1, \dots, X_r$ *be generators of a subspace* V *of* $\mathbb{R}^n$*. Let* W *be the set of all elements in* $\mathbb{R}^n$ *which are perpendicular to* $X_1, \dots, X_r$*. Show that* $W = V^\perp$*.*

Exercise 4 *Show that the set of all real polynomials is a subspace of the vector space of all real and continuous functions on* $\mathbb{R}$*. Exhibit a generating family for this subspace.*

Exercise 5 *Let* V *be a vector space over a field* F*. Show that any subspace of* V *is itself a vector space.*

Exercise 6 *To gauge the complexity of a computational task, one can count the number of elementary operations (additions, subtractions, multiplications and divisions) required. For a rough count, one can consider multiplication and divisions only, referring to those jointly as* multiplicative operations*. Start by considering a* 2 *by* 2 *invertible matrix* $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ *and check that* 8 *multiplicative operations are necessary for inverting this matrix by using the Gauss elimination technique.*

(i) How many multiplicative operations are necessary for inverting a 3×3 *matrix by the same technique ?*

(ii) What about a $n \times n$ *matrix ?*

(iii) If a very slow computer needs 1 second to invert a 3×3 *matrix, how long will it take to invert a* 12×12 *matrix ?*