Calculus I

Quiz 9

Name: ME

Explain your solution process clearly.
Write legible.

1. [4 points] Determine whether the statement is true or false. If it is true, explain why. If it is false, give an example that disproves the statement.

(a) The inverse function of $y = e^{x/4}$ is $y = 4\ln(x)$.

TRUE, since $4\ln(e^{x/4}) = 4\cdot\left(\frac{x}{4}\right)\ln(e) = x$ for all x,

and $e^{\frac{1}{4}(4\ln x)} = e^{\ln(x)} = x$ for all $x > 0$.

(b) $\int_1^3 f'(t)dt = f(3) - f(1)$.

The Fundamental Theorem of Calculus says

$\int_a^b g(t)dt = G(b) - G(a)$ for any G satisfying $G' = g$.

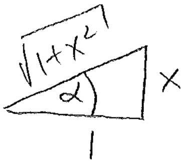
Here: $G = f$, then $G' = f' = g$. So the statement is true.

2. [3 points] Find the derivative of $\arctan(x)$ using that the derivative of $\tan(x)$ is $(\sec(x))^2$.

$(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))} \Rightarrow (\arctan(x))' = \dfrac{1}{(\sec(\arctan(x)))^2}$

$\Downarrow$

$\alpha = \arctan x \Rightarrow \tan\alpha = x$



$\Rightarrow \sec(\alpha) = \sqrt{1+x^2} \longrightarrow (\arctan(x))' = \dfrac{1}{1+x^2}$

3. [3 points] Find $\lim_{x\to 0+} x\ln(x)$.

$x \to 0$, $\ln(x) \to -\infty \Rightarrow$ consider $x \cdot \ln(x) = \dfrac{\ln(x)}{\frac{1}{x}}$, $\ln(x) \to -\infty$, $\frac{1}{x} \to 0^+$

$\lim_{x\to 0^+} \dfrac{\ln(x)}{\frac{1}{x}} = \lim_{x\to 0^+} \dfrac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} -x = 0$

L'Hospital's $\Rightarrow$ rule

$\lim_{x\to 0^+} x\cdot\ln(x) = 0.$