Calculus I

Quiz 4

Name: ME

Explain your solution process clearly.
Write legible.

1. (4 points) Consider the function $f(x) = \frac{x+1}{x-1}$.

   (a) Find the domain of f. The functions $h(x)=x+1$ and $g(x)=x-1$ are defined for all x. However, $g(1)=0$. Hence, the domain of f is all real numbers except 1.

   (b) Find the horizontal asymptotes of f (if there are any).

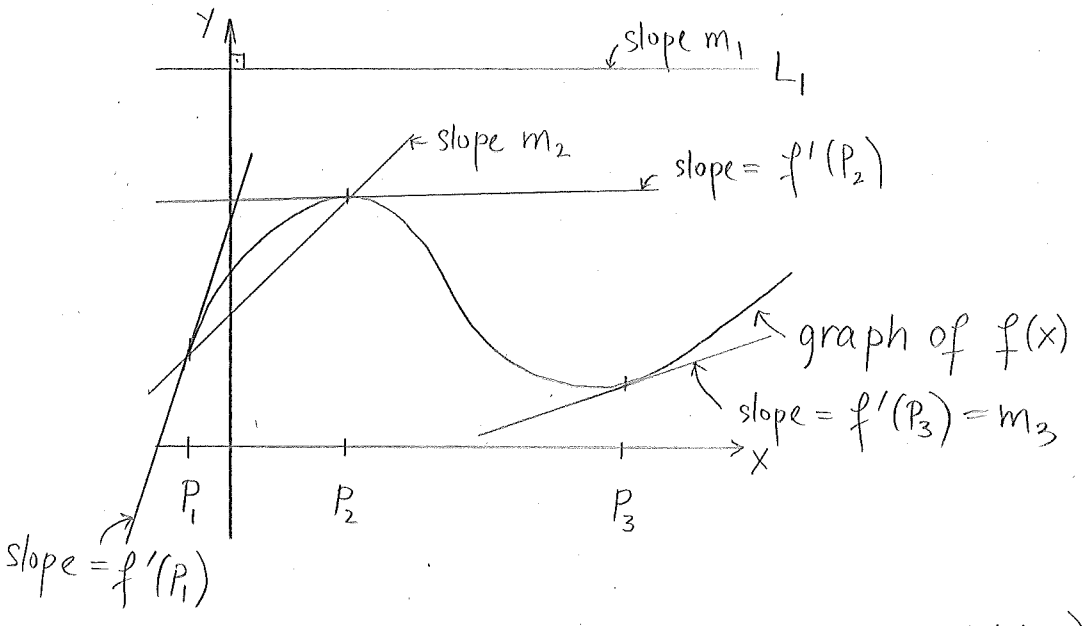
$$\lim_{x\to\infty} \frac{x+1}{x-1} = \lim_{x\to\infty} \frac{x\left(1+\frac{1}{x}\right)}{x\left(1-\frac{1}{x}\right)} = \lim_{x\to\infty} \frac{1+\left(\frac{1}{x}\right)}{1-\left(\frac{1}{x}\right)} \to 0 \to 0 = \frac{1}{1} = 1, \quad \lim_{x\to-\infty} \frac{x+1}{x-1} = \lim_{x\to-\infty} \frac{1+\frac{1}{x}}{1-\frac{1}{x}} = 1$$

   f has only 1 horizontal asymptote, it is: $y=1$.

   (c) Find the vertical asymptotes of f (if there are any).

Since $\lim_{x\to 1^-} \frac{x+1}{x-1} = -\infty$ (and $\lim_{x\to 1^+} \frac{x+1}{x-1} = \infty$), it follows that $x=1$ is a vertical asymptote of f.

2. (6 points) Consider the graphs given below. Let m_1 be the slope of L_1, m_2 the slope of the line which passes through $(P_1, f(P_1))$ and $(P_2, f(P_2))$, and m_3 the slope of the line tangent to the graph of f at $(P_3, f(P_3))$. Order the numbers $f'(P_1)$, $f'(P_2)$, m_1, m_2 and m_3 from smallest to largest.



$$m_1 = f'(P_2) < m_3 < m_2 < f'(P_1)$$