

Quiz 1

Name: _____

Explain your solution process clearly.
Write legible.

1. (5 points) Give the definition of a function.

A function is a rule which assigns each point from a set A to exactly one point in a set B . One could also draw a picture, like an arrow diagram, to support one's written definition.

Of course there are other ways to write this. For instance: A function is the particular association of every element from one set to exactly one element each of another set.

2. (5 points) Consider the function
- $y(x) = x^2 - \frac{5}{6}x - 1$
- .

- (a) With respect to which vertical line is the graph of
- $y(x)$
- symmetric? Hint: Complete the square first.

Let us complete the square first:

$$\begin{aligned} y &= x^2 - \frac{5}{6}x - 1 \\ &= x^2 - 2\frac{5}{12}x + \frac{25}{144} - \frac{25}{144} - 1 \\ &= \left(x - \frac{5}{12}\right)^2 - \frac{169}{144}. \end{aligned}$$

Since $y(x + 5/12) = x^2 - 169/144 = y(-x + 5/12)$ it follows that the graph of $y(x)$ is symmetric with respect to the line given by the equation $x = 5/12$ in the xy plane.

Or: compute the roots of $y(x)$. That is, find the 2 values for x such that

$$\begin{aligned} 0 &= y(x) = \left(x - \frac{5}{12}\right)^2 - \frac{169}{144} \\ \Rightarrow x - \frac{5}{12} &= \pm \frac{13}{12} \Rightarrow x = \frac{5}{12} \pm \frac{13}{12}. \end{aligned}$$

Hence $y(x) = 0$ when $x = 3/2$ or when $x = -2/3$. The graph of $y(x)$ is a parabola which implies that the vertical with respect to which the graph of $y(x)$ is symmetric goes through the midpoints of the two zeros. The distance between the two zeros is $3/2 + 2/3 = 13/6$, so the midpoint is $3/2 - 13/12 = 5/12$. This leads to the same line as described above.

- (b) With respect to which horizontal line is the graph of
- $y(x)$
- symmetric? There is no horizontal line with respect to which the graph of
- $y(x)$
- is symmetric as this would contradict the vertical lines test.