
Homework n^{o} 9

Exercise 1. By using elementary row operations, find the inverse for the following matrices :

$$a)\ \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 2 & 7 \end{pmatrix} \quad b)\ \begin{pmatrix} 2 & 1 & 2 \\ 0 & 3 & -1 \\ 4 & 1 & 1 \end{pmatrix} \quad c)\ \begin{pmatrix} 2 & 4 & 3 \\ -1 & 3 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

Exercise 2. Consider the equation

$$\begin{aligned} x + 2y + 3z &= 4 \\ x + ky + 4z &= 6 \\ x + 2y + (k+2)z &= 6 \end{aligned}$$

where k is an arbitrary constant.

1. For which values of k does this system have a unique solution ?
2. When is there no solution ?
3. When are there infinitely many solutions ?

Exercise 3. A conic is a curve in $\mathbb{R}^2$ that can be described by an equation of the form

$$f(x,y) = c_1 + c_2x + c_3y + c_4x^2 + c_5xy + c_6y^2 = 0,$$

where at least one of the coefficients c_i is non-zero. Find the conic passing through the following points.

i) $(0,0)$, $(1,0)$, $(0,1)$, $(1,1)$.

ii) $(0,0)$, $(1,0)$, $(2,0)$, $(3,0)$, $(1,1)$.

Exercise 4. Let $\mathcal{A} \in M_{mn}(\mathbb{R})$ and $X = {}^T(x_1,\dots,x_n) \in \mathbb{R}^n$. The columns of $\mathcal{A}$ are denoted by $\mathcal{A}^1,\dots,\mathcal{A}^n$, while the rows of $\mathcal{A}$ are denoted by $\mathcal{A}_1,\dots,\mathcal{A}_m$. Show that the following three statements are equivalent :

1. $\mathcal{A}X = 0$,
2. the vector $\overrightarrow{0X}$ is perpendicular to the vector ${}^T\mathcal{A}_j$, for each $j \in \{1,\dots,m\}$,
3. the following linear relation holds :

$$x_1\mathcal{A}^1 + x_2\mathcal{A}^2 + \cdots + x_n\mathcal{A}^n = 0.$$