University of Nagoya, G30 program Autumn 2013

Mathematics Tutorial I (linear algebra part) Instructor : Serge Richard

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Homework n^o 6

Exercise 1. For any $\theta \in \mathbb{R}$, recall that the matrix

$$R(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$

represents a rotation by θ in $\mathbb{R}^2$.

1. For ${}^TX = (1, 2)$, what are its coordinates after a rotation of $\pi/4$?
2. For ${}^TY = (-1, 3)$, what are its coordinates after a rotation of $\pi/2$?

Draw a picture of your results.

Exercise 2. Let

$$\mathcal{A} = \begin{pmatrix} 2 & 3 & -1 & 1 \\ 1 & 4 & 2 & -2 \\ -1 & 1 & 3 & -5 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

and let $\mathcal{U}$ be one of the matrices shown below. Compute $\mathcal{U}\mathcal{A}$.

$$a)\ \mathcal{U} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad b)\ \mathcal{U} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad c)\ \mathcal{U} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$d)\ \mathcal{U} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad e)\ \mathcal{U} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad f)\ \mathcal{U} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Exercise 3. Do the same exercise with the following matrices $\mathcal{U}$ and $\mathcal{A}$ as above :

$$a)\ \mathcal{U} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad b)\ \mathcal{U} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad c)\ \mathcal{U} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 5 & 0 & 1 \end{pmatrix} \quad d)\ \mathcal{U} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Exercise 4. Let $\mathcal{A} \in M_{mn}(\mathbb{R})$. For $r \in \{1, \ldots, m\}$ and $s \in \{1, \ldots, m\}$, let $I_{rs} \in M_{mm}(\mathbb{R})$ be the matrix whose rs-component is 1 and all the other ones are equal to 0. Answer the following questions with words :

1. What is $I_{rs}\mathcal{A}$?
2. For $r \neq s$, what is $(I_{rs} + I_{sr})\,\mathcal{A}$?
3. For $r \neq s$, what is $(\mathbb{I}_m + I_{rs} + I_{sr} - I_{rr} - I_{ss})\,\mathcal{A}$?
4. For $r \neq s$, what is $(\mathbb{I}_m + cI_{rs})\,\mathcal{A}$, for some $c \in \mathbb{R}$?

Exercise 5. Find a non-trivial solution for each of the following systems of equations.

$$a) \quad \begin{aligned} 2x - 3y + 4z &= 0 \\ 3x + y + z &= 0 \end{aligned}$$

$$b) \quad \begin{aligned} 2x + y + 4z + w &= 0 \\ -3x + 2y - 3z + w &= 0 \\ x + y + z &= 0 \end{aligned}$$

$$c) \quad \begin{aligned} -2x + 3y + z + 4w &= 0 \\ x + y + 2z + 3w &= 0 \\ 2x + y + z - 2w &= 0 \end{aligned}$$