University of Nagoya, G30 program Autumn 2013

Mathematics Tutorial I (linear algebra part) Instructor : Serge Richard

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Homework n° 3

Exercice 1. Let $P, N \in \mathbb{R}^n$ with $N \neq 0$, and let $\lambda \in \mathbb{R}^*$. Let $H_{P,N}$ denote the hyperplane passing through P and normal to N. Show that

1. $H_{P,N} = H_{P,\lambda N}$
2. if $P' \in H_{P,N}$, then $H_{P',N} = H_{P,N}$

Exercice 2. Determine the cosine of the angle between the two planes defined by

$$\{(x,y,z) \in \mathbb{R}^3 \mid 2x - y + z = 0\} \quad \text{and} \quad \{(x,y,z) \in \mathbb{R}^3 \mid x + 2y - z = 1\}.$$

Same question for the planes defined by

$$\{(x,y,z) \in \mathbb{R}^3 \mid x = 1\} \quad \text{and} \quad \{(x,y,z) \in \mathbb{R}^3 \mid 3x + 2y - 7z = 1\}.$$

Exercice 3. Find the equation of the plane in $\mathbb{R}^3$ passing through the three points $P_1 = (1,2,-1)$, $P_2 = (-1,1,4)$ and $P_3 = (1,3,-2)$.

Exercice 4. Let $P = (-1,1,7)$, $Q = (1,3,5)$ and $N = (-1,1,-1)$. Determine the distance between the point Q and the plane $H_{P,N}$.

Exercice 5. Let $P = (1,1,1)$, $Q = (1,-1,2)$ and $N = (1,2,3)$. Find the intersection of the line through Q and having the direction N with the plane $H_{P,N}$.

Exercice 6. Determine the equation of the hyperplane in $\mathbb{R}^4$ passing through the point $(1,1,1,1)$ and which is parallel to the hyperplane defined by

$$\{(x_1,x_2,x_3,x_4) \in \mathbb{R}^4 \mid 1x_1 + 2x_2 + 3x_3 + 4x_4 = 5\}.$$

Similarly, for any $n > 1$ determine the equation of the hyperplane in $\mathbb{R}^n$ passing through the point $(1,1,\dots,1)$ and which is parallel to the hyperplane defined by

$$\big\{(x_1,x_2,\dots,x_n) \in \mathbb{R}^n \mid \sum_{j=1}^{n} j\,x_j = n+1\big\}.$$

Does something special happen for $n = 2$?