Homework n^o 12

Exercise 1. Determine the rank of the following matrices :

$$a)\ \begin{pmatrix} 6 & 3 & -4 \\ -4 & 1 & -6 \\ 1 & 2 & -5 \end{pmatrix} \quad b)\ \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix} \quad c)\ \begin{pmatrix} 0 & 1 & 3 & -2 \\ 2 & 1 & -4 & 3 \\ 2 & 3 & 2 & -1 \end{pmatrix} \quad d)\ \begin{pmatrix} 1 & 2 & -1 & 2 & 1 \\ 2 & 4 & 1 & -2 & 3 \\ 3 & 6 & 2 & -6 & 5 \end{pmatrix}$$

Exercise 2. Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be the map defined by $F(x, y) = (2x, 3y)$ for any $(x, y) \in \mathbb{R}^2$. Describe the image by F of the points lying on the unit circle centered at $(0, 0)$, *i.e.* $\big\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\big\}$.

Exercise 3. Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be the map defined by $F(x, y) = (xy, y)$ for any $(x, y) \in \mathbb{R}^2$. Describe the image by F of the line $\{(x, y) \in \mathbb{R}^2 \mid x = 2\}$.

Exercise 4. Let V be a vector space of dimension n, and let $\{X_1, \dots, X_n\}$ be a basis for V. Let F be a linear map from V into itself. Show that F is uniquely defined if one knows $F(X_j)$ for $j \in \{1, \dots, n\}$. Is it also true if F is an arbitrary map from V into itself ?

Exercise 5. Let V, W be vector spaces over the same field, and let $T : V \to W$ be a linear map. Show that the following set is a subspace of V :

$$\{x \in V \mid T(x) = 0\}.$$

This subspace is call the kernel of T.

Exercise 6. Show that the image of a convex set under a linear map is a convex set.