Calculus I

Final Exam

Name: _____

To receive full credit you need to show your work. That is, full credit is only given if you explain your thought process clearly and in detail.

This final exam contributes 40% to your final grade in Calculus I.

1. [10 points] Evaluate the integrals.

(a) $\int \frac{x}{1+x^2}\, dx$

(b) $\int_0^1 3^x x\, dx$

(c) $\int \frac{1}{2+3x^2}\, dx$

(d) $\int_0^2 |2x-1|\, dx$

(e) If $f(x)$ is the slope of a trail at a distance of x kilometers from the start of the trail, what does $\int_3^5 f(x)\, dx$ represent?

2. [10 points] Find the limits (you may use L'Hospital's rule when possible).

(a) $\lim_{x\to 0^+} \frac{\cos(x)}{x}$

(b) $\lim_{x\to 0^+} x^{\sqrt{x}}$

(c) $\lim_{x\to 0^-} \tan^{-1}(1/x)$

(d) $\lim_{x\to 1+} \frac{x}{x-1} - \frac{1}{\ln(x)}$

(e) $\lim_{x\to 0} x\sin(1/x)$

3.[10 points] Where should the point P be chosen on the line segment AB so as to maximize the angle θ?

A diagram was included in the distributed version of the final exam (a vertical line from a point A to a point B, P some point on that line segment, a horizontal line starting at point A going 2 units to the right ending at point C, a horizontal line starting at point B going 3 units to the right ending at point D, theta is angle CPD)

4. [10 points] Determine whether the statement is true or false. If it is true, explain why. If it is false, explain why or, if possible, give an example that disproves the statement.

(a) If f is one-to-one and differentiable, then $(f^{-1})'(x) = 1/f'(x)$ for all x in the domain of f^{-1}.

(b) $\arctan(x) = \frac{\sin^{-1}(x)}{\cos^{-1}(x)}$

(c) All continuous functions are differentiable.

(d) $\frac{d}{dx}(\tan^2(x)) = \frac{d}{dx}(\sec^2(x))$

(e) If a function f has a vertical asymptote at 1, then f is not defined at 1.

5. [10 points] To sketch the curve of $g(x) = \frac{x}{1-x^2}$ (see part (h)), first determine the following properties of

(a) Domain of g

(b) Intercepts of g

(c) Symmetry of g

(d) Asymptotes of g

(e) Interval of Increase or Decrease of g

(f) Local Maximum and Minimum Values of g

(g) Concavity and Points of Inflection of g

(h) Sketch the graph of g