ABSTRACTS

Toshiaki Adachi (*Nagoya Institute of Technology*) . January 28th (Mon), 15:30–16:15

Kähler manifolds and extrinsic shapes of curves through isometric immersions

On a Kähler manifold we have natural uniform magnetic fields which are called Kähler magnetic fields. Trajectories of charged particles under such magnetic fields can be considered as generalized objects of geodesics. In this talk I study Kähler manifolds, especially complex space forms, by extrinsic shapes of trajectories through isometric immersions into real space forms. Since trajectories are typical examples of curves of order 2, I also make mention of isometric immersions which preserve curvature logarithmic derivatives of curves of order 2. Here, curvature logarithmic derivative of a curve is the ratio of the derivative of the first geodesic curvature to itself.

Kazuo Akutagawa (*Tokyo University of Science*) . . . . January 26th (Sat), 16:30–17:15

On the Yamabe invariant of $M \times S^1$

Let M be a closed smooth manifold of dimension n (≥ 2). The relationship between the *Yamabe invariant* $Y(M)$ of M and $Y(M \times S^1)$ is very interesting, as well as *Surgery Theory* for the Yamabe invariant. The expectation is the following :

Conjecture (1) Assume that $Y(M) \leq 0$. Then,

$$Y(M \times S^1) = 0.$$

(2) Assume that $Y(M) > 0$. Then,

$$Y(M \times S^1) \leq \Big(Y(M)/Y(S^n)\Big)^{n/n+1} Y(S^{n+1}).$$

Moreover, if M admits a *supreme* positive Einstein metric $\hat{g}$, that is, $Y(M, [\hat{g}]) = Y(M)$, then

$$Y(M \times S^1) = \Big(Y(M)/Y(S^n)\Big)^{n/n+1} Y(S^{n+1}).$$

When $Y(M) \leq 0$, the conjecture is reduced to the following problem :
“ Does the non-positivity $Y(M) \leq 0$ imply that $Y(M \times S^1) \leq 0$? ”
Hence, the first conjecture (1) has been solved affirmatively for certain large classes of manifolds. For instance, *enlargeable manifolds*.

In this talk, we will concentrate on the second conjecture (2). I will explain its background and describe recent progress on it.

Bohui Chen (*Sichuan University*) . . . . January 26th (Sat), 13:30–14:15

Virtual manifolds and localization

This is a joint work with Tian. In this talk, we explore the virtual technique that is very useful in studying moduli problem from differential geometric point of view. We introduce a class of new objects ”virtual manifolds/orbifolds”, on which we develop the integration theory. In particular, the virtual localization formula is obtained.

Daguang Chen (_Tsinghua University_) . . . . January 29th (Tue), 10:30–11:15

Extrinsic estimates for eigenvalues of the Dirac operator

In this talk, I will discuss extrinsic inequalities for eigenvalues of the Dirac operator. For a compact spin manifold isometrically embedding into Euclidean space, we derive the extrinsic inequalities for the square of the Dirac operator. Further, from the recursion formula of Cheng-Yang [Math. Ann., **337** (2007), 159-175], we also obtain the ratios of eigenvalues, which is best possible in the meaning of the order. For a compact complex spin manifold with a holomorphic isometric embedding into the complex projective space, the corresponding results are also obtained.

Qun Chen (_Wuhan University_) . . . . January 27th (Sun), 10:30–11:15

Dirac-harmonic Maps and Dirac equations on Riemann surfaces

Dirac-harmonic maps are critical points of a functional in which the energy of maps between Riemannian manifolds is coupled with a Dirac term of some twisted spinor bundle. In this talk, we will introduce our recent results on this model as well as related nonlinear Dirac equations on Riemann surfaces.

Huitao Feng (_Nankai University_) . . . . January 28th (Mon), 9:30–10:15

On the eta invariants of Atiyah-Patodi-Singer

In this talk, a geometric proof of Bismut-Zhang localization formula for eta-invariants is given.

Zhen Guo (_Yunnan Normal University_) . . . . January 29th (Tue), 9:30–10:15

Shuichi Izumiya (_Hokkaido University_) . . . . January 27th (Sun), 13:30–14:15

Legendrian dualities for “flat” surfaces in Lorentz-Minkowski pseudo-spheres I

The duality concepts we use here are those introduced in [1], where four Legendrian double fibrations are considered on the subsets Δ_i, $i = 1,\ldots,4$, of the product of two of the pseudo spheres $H^n(-1)$, S^n_1 and LC^* in Lorentz-Minkowski space as follows:

(1) (a) $H^n(-1)\times S^n_1 \supset \Delta_1 = \{(v,w) \mid \langle v,w\rangle = 0\}$,
(b) $\pi_{11} : \Delta_1 \to H^n(-1)$, $\pi_{12} : \Delta_1 \to S^n_1$,
(c) $\theta_{11} = \langle dv,w\rangle|\Delta_1$, $\theta_{12} = \langle v,dw\rangle|\Delta_1$.

(2) (a) $H^n(-1)\times LC^* \supset \Delta_2 = \{(v,w) \mid \langle v,w\rangle = -1\}$,
(b) $\pi_{21} : \Delta_2 \to H^n(-1)$,$\pi_{22} : \Delta_2 \to LC^*$,
(c) $\theta_{21} = \langle dv,w\rangle|\Delta_2$, $\theta_{22} = \langle v,dw\rangle|\Delta_2$.

(3) (a) $LC^*\times S^n_1 \supset \Delta_3 = \{(v,w) \mid \langle v,w\rangle = 1\}$,
(b) $\pi_{31} : \Delta_3 \to LC^*$,$\pi_{32} : \Delta_3 \to S^n_1$,
(c) $\theta_{31} = \langle dv,w\rangle|\Delta_3$, $\theta_{32} = \langle v,dw\rangle|\Delta_3$.

(4) (a) $LC^*\times LC^* \supset \Delta_4 = \{(v,w) \mid \langle v,w\rangle = -2\}$,
(b) $\pi_{41} : \Delta_4 \to LC^*$,$\pi_{42} : \Delta_4 \to LC^*$,

(c) $\theta_{41} = \langle dv, w\rangle|\Delta_4$, $\theta_{42} = \langle v, dw\rangle|\Delta_4$.

Above, $\pi_{i1}(v,w) = v$ and $\pi_{i2}(v,w) = w$ for $i = 1,\dots,4$, $\langle dv, w\rangle = -w_0 dv_0 + \sum_{i=1}^n w_i dv_i$ and $\langle v, dw\rangle = -v_0 dw_0 + \sum_{i=1}^n v_i dw_i$. The 1-forms $\theta_{i1}^{-1}(0)$ and $\theta_{i2}^{-1}(0)$, $i = 1,\dots,4$, define the same tangent hyperplane field over Δ_i which is denoted by K_i. The duality theorem in [1] asserts that the pairs (Δ_i, K_i), $i = 1,\dots,4$, are contact manifolds and π_{i1} and π_{i2} are Legendrian fibrations.

On the other hand, there is a notion of *linear Weingarten surfaces* (in short, *LW-surfaces*) in each of $H^3(-1)$ and S_1^3. This class of surfaces contains very important surfaces such as CMC-1 surfaces and (intrinsically) flat surfaces. These surfaces are the members of a generic subclass of the LW-surfaces called of Bryant type for $H_+^3(-1)$ (resp., of Bianchi type for S_1^3). There are Enneper-Weierstrass type representation formulae for such surfaces. The horo-flat surfaces are in the exceptional class (i.e., non-Bryant type) of LW-surfaces such that we have no Enneper-Weierstrass type representation formulae[2]. In this talk, we will explain "mandala" of the above Legendrian dulaities and the Δ_1-dual relation between LW-surfaces in $H_+^3(-1)$ and S_1^3. We can show that the LW-surfaces of Bryant type (resp., horo-flat surfaces) is corresponding to the LW-surfaces of Bianchi type (resp., de Sitter horo-flat surfaces). This is a joint work with Kentaro Saji.

References

[1] S. Izumiya, Legendrian dualities and spacelike hypersurfaces in the lightcone. Preprint, 2005.

[2] S. Izumiya, K. Saji and M. Takahashi, *Horospherical flat surfaces in hyperbolic* 3*-space*. Preprint 2007.

Miyuki Koiso (*Nara Women's University*) . . . . January 28th (Mon), 13:30–14:15

A free boundary problem for surfaces with constant anisotropic mean curvature

Minimal surfaces and surfaces with constant mean curvature are probably the most fundamental subjects of research in variational problems for surfaces. They are critical points of the area functional, critical points of the area functional for volume-preserving variations, respectively. Sometimes it is said that they serve as a model of soap films, soap bubbles, respectively, for the free surface energy of liquid is regarded as isotropic and if the liquid is homogeneous, it is proportional to the surface area. However, if the material is a crystalline solid or a liquid crystal, we need to consider an anisotropic surface energy, that is, one that depends on the direction of the surface.

Let $F : S^2 \to \mathbf{R}^+$ be a positive, smooth function. For a smooth, oriented immersed surface $X : \Sigma = \Sigma^2 \to \mathbf{R}^3$ whose Gauss map is $\nu : \Sigma \to S^2$, we define the functional

$$\mathcal{F}(X) := \int_\Sigma F(\nu)\, d\Sigma\,,$$

where $d\Sigma$ is the area element of X. Such functional is used to model anisotropic surface energies. Applications can be found in many branches of the physical sciences including metallurgy and crystallography. If the functional satisfies a certain convexity condition, it is called a constant coefficient parametric elliptic functional.

Each equilibrium surface (that is, a critical point) for a constant coefficient parametric elliptic functional with a volume constraint has a constant anisotropic mean curvature. As a special case, we have surfaces with constant mean curvature. In this talk, we mainly study equilibrium surfaces for such a functional having free boundaries supported in given horizontal planes. A wetting energy term for the surface to plane interface is included. An equilibrium surface is said to be stable if the second variation of the energy functional is nonnegative for all volume-preserving variations of the surface which satisfy the boundary conditions. For a large class of rotationally symmetric energy functionals, the existence and uniqueness for stable solutions is proved, and moreover the geometric properties of the solution are determined.

The results which are discussed here come from recent joint work of Bennett Palmer and the author.

Anmin Li (*Sichuan University*) . . . . January 28th (Mon), 14:30–15:15

Some new results for 4-th order PDE in Differential Geometry

We study the equation

$$\sum_{i,j=1}^{n} U^{ij} w_{ij} = -L \,, \quad w = |\det D^2 f|^a \,, \tag{1}$$

where $a \neq 0$ is a constant, (U^{ij}) the cofactor matrix of the Hessian matrix $D^2 f$ of a smooth convex function f defined on a convex domain $\Omega \subset \mathbf{R}^n$, L a given function defined in Ω. When $a = -\frac{n+1}{n+2}$, the PDE (1) is the prescribed affine mean curvature equation; when $a = -1$ the PDE (1) is called the Abreu equation, which appears in the study of the differential geometry of toric varieties (see [A], [D-1], [D-2], [D-3]), where L is the scalar curvature of the Kähler metric. We will talk about some of our new results in this direction.

Haizhong Li (*Tsinghua University*) . . . . January 27th (Sun), 11:30–12:15

Variational problems for geometry of submanifolds

In this talk, we present a survey of some variational problems in geometry of submanifolds, which includes our recent research results in geometry of r-minimal submanifolds (see [CL]) and variations of some parametric elliptic functional. We introduce the concepts of r-minimal submanifolds for n-dimensional submanifolds in an $(n+p)$-dimensional space form $R^{n+p}(c)$ and study the stability of compact r-minimal submanifolds in the unit sphere S^{n+p}. We consider a variation problem concerning certain parametric elliptic functional and collect some related results. We state integral formula of Minkowski's type (see [HL]) and new characterizations of the Wulff shape. We also propose some related problems at the end of talk.

[CL] L. F. Cao and H. Li, r-minimal submanifolds in space forms, Ann. Global Anal. Geom., 32(2007), 311-341.

[HL] Y. J. He and H. Li, Integral formula of Minkowski type and new characterization of the Wulff shape, arXiv:math.DG/0703187, 2007.

Huili Liu (*Northeastern University*) . . . . January 28th (Mon), 11:30–12:15

Curves and surfaces in lightlike cone

In this talk, we concern with the curves, surfaces and hypersurfaces in $(n+1)$ dimensional lightlike cone. We will give the fundamental theories and some properties of such curves, surfaces and hypersurfaces and characterize some of them as the following.

Curves and surfaces

Theorem 1 *Let $x : \mathbf{I} \to \mathbf{Q}^{n+1} \subset \mathbf{E}_1^{n+2}$ $(n \geq 2)$ be a Frenet curve in $\mathbf{Q}^{n+1}$ with the induced arc length parameter s. Then*

$$\sqrt{< x''', x''' > - < x'', x'' >^2} < \mathrm{d}x, \mathrm{d}x >$$

is a conformal invariant.

Theorem 2 *Let $x : \mathbf{I} \to \mathbf{Q}^2$ be a curve with the cone curvature function $\kappa = cs^{-2}$ for some nonzero constant c and the arc length parameter s, then $x(s)$ can be written as the following:*

(i)
$$x(s) = a_1 s + a_2 s^{(1+\sqrt{1+2c})} + a_3 s^{(1-\sqrt{1+2c})}$$

for $c \neq -\frac{1}{2}$, where a_1, a_2, $a_3 \in \mathbf{E}_1^3$;

(ii)
$$x(s) = a_1 s + a_2 s \log s + a_3 s \log^2 s$$

for $c = -\frac{1}{2}$, where a_1, a_2, $a_3 \in \mathbf{E}_1^3$.

Theorem 3 *Let $x : \mathbf{I} \to \mathbf{Q}^2$ be a curve with the cone curvature function $\kappa = \text{constant}$ and the arc length parameter s, then $x(s)$ can be written as the following: (a_1, a_2, $a_3 \in \mathbf{E}_1^3$)*

(i)
$$x(s) = a_1 s^2 + a_2 s + a_3$$

for $\kappa = 0$, the curve is a parabola;

(ii)
$$x(s) = a_1 \sinh(\sqrt{2\kappa})s + a_2 \cosh(\sqrt{2\kappa})s + a_3$$

for $\kappa > 0$, the curve is a hyperbola;

(iii)
$$x(s) = a_1 \sin(\sqrt{-2\kappa})s + a_2 \cos(\sqrt{-2\kappa})s + a_3$$

for $\kappa < 0$, the curve is an ellipse.

Theorem 4 *Let $x : \mathbf{I} \to \mathbf{Q}^2$ be a curve whose tangent vector field intersects a fixed vector in $\mathbf{E}_1^3$ at a constant angle (helix), then $x(s)$ can be written as the curves given by Theorem 2 and Theorem 3.*

Theorem 5 *Let $x : \mathbf{M} \to \mathbf{Q}^3$ be a homogeneous surface. Then it is congruent to one of the following surfaces:*

(i) a totally umbilical surface;

(ii) $x(u, v) = a(\sin u, \cos u, \sinh v, \cosh v)$ where $0 \neq a \in \mathbf{R}$.

Theorem 6 *Let $x : \mathbf{M} \to \mathbf{Q}^4$ be a homogeneous surface with vanishing 1-form $\Omega = \mu e \mathrm{dz}$. Then it can be written as one of the following surfaces:*

(i) $x(u, v) = a_1 + a_2 u + a_3 u^2 + a_4 \sin v + a_5 \cos v$;

(ii) $x(u, v) = a_1 + a_2 \sinh u + a_3 \cosh u + a_4 \sin v + a_5 \cos v$;

(iii) $x(u, v) = a_1 + a_2 \sin u + a_3 \cos u + a_4 \sin v + a_5 \cos v$,

where a_1, a_2, a_3, a_4, $a_5 \in \mathbf{E}_1^5$.

Hypersurfaces

The structure equations for the hypersurface $x:\mathbf{M}^n \to \mathbf{Q}^{n+1} \hookrightarrow \mathbf{E}_1^{n+2}$:

$$
\begin{aligned}
< e_i(x), e_j(x) > &= g_{ij} = g_{ji}, && (2)\\
e_i(y) &= A_{ij} g^{j\alpha} e_\alpha(x), && (3)\\
e_j(e_i(x)) &= \Gamma^k_{ij} e_k(x) - A_{ji} x - g_{ij} y. && (4)
\end{aligned}
$$

Therefore, the integrability conditions of x are:

$$
\begin{aligned}
A_{ij} &= A_{ji}, && (5)\\
A_{ij,k} &= A_{ik,j}, && (6)\\
R_{ijkl} &= A_{ik} g_{jl} + A_{jl} g_{ik} - A_{jk} g_{il} - A_{il} g_{jk}. && (7)
\end{aligned}
$$

From (7) we get the Ricci curvature tensor R_{ij} and the normalized scalar curvature κ as the follows:

$$
\begin{aligned}
R_{ik} &= (n-2)A_{ik} + (\mathrm{trace}A) g_{ik}, && (8)\\
n(n-1)\kappa &= 2(n-1)(\mathrm{trace}A). && (9)
\end{aligned}
$$

Theorem 7 *Let $x:\mathbf{M}^n \to \mathbf{Q}^{n+1} \hookrightarrow \mathbf{E}_1^{n+2}$ be any nondegenerate hypersurface in lightlike cone $\mathbf{Q}^{n+1}$, $n>2$, then x is conformal flat.*

Theorem 8 *Let $x:\mathbf{M}^n \to \mathbf{Q}^{n+1} \hookrightarrow \mathbf{E}_1^{n+2}$ be a nondegenerate hypersurface. If $A_{ij} = \lambda g_{ij}$ for some $\lambda \in C^\infty(\mathbf{M})$, then x lies in a hyperquadric.*

Theorem 9 *Let $x:\mathbf{M}^n \to \mathbf{Q}^{n+1} \hookrightarrow \mathbf{E}_1^{n+2}$ be a nondegenerate hypersurface,* $\mathrm{rank}(A) = n$. *If $n=2$, then x is maximal if and only if the associated hypersurface y of x is also maximal.*

Finally, using the moving frame method, we have the first and the second variation formulas of the area integral for the hypersurface in lightlike cone $x:\mathbf{M}^n \to \mathbf{Q}^{n+1} \hookrightarrow \mathbf{E}_1^{n+2}$:

$$
\begin{aligned}
V'(0) &= \frac{\mathrm{d}}{\mathrm{d}t}\mathrm{Vol}(\mathbf{M}_t)\ |_{t=0} && (10)\\
&= \frac{\partial}{\partial t}\int_{\mathbf{M}} \omega^1 \wedge \cdots \wedge \cdots \wedge \omega^n\ |_{t=0}\\
&= \int_{\mathbf{M}} \frac{\partial}{\partial t}(\omega^1 \wedge \cdots \wedge \cdots \wedge \omega^n)\ |_{t=0}\\
&= n\int_{\mathbf{M}} H\langle V^{\mathrm{N}}, y\rangle \mathrm{d}\mathbf{M}.
\end{aligned}
$$

Theorem 10 *The area variation of a hypersurface $x:\mathbf{M}^n \to \mathbf{Q}^{n+1} \hookrightarrow \mathbf{E}_1^{n+2}$ depends only on the normal component of the variation vector field. The critical hypersurfaces for the area integral are exactly the hypersurfaces with vanishing mean curvature H.*

Theorem 11 *Let $x:\mathbf{M}^n \to \mathbf{Q}^{n+1} \hookrightarrow \mathbf{E}_1^{n+2}$ be a hypersurface with $H \equiv 0$.The second variation formula of the area integral for x is*

$$
\begin{aligned}
V''(0) &= \frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathrm{Vol}(\mathbf{M}_t)\Big|_{t=0} && (11)\\
&= \int_{\mathbf{M}} \frac{\partial}{\partial t}(nH)\Big|_{t=0} \langle V^{\mathrm{N}}, y\rangle \mathrm{d}\mathbf{M}\\
&= \int_{\mathbf{M}} \{\langle V^{\mathrm{N}}, y\rangle \Delta \langle V^{\mathrm{N}}, y\rangle - (\langle V^{\mathrm{N}}, y\rangle)^2 ||A||^2\} \mathrm{d}\mathbf{M}.
\end{aligned}
$$

Therefore, the hypersurface $x:\mathbf{M}^n \to \mathbf{Q}^{n+1} \hookrightarrow \mathbf{E}_1^{n+2}$ with $H \equiv 0$ satisfies $V''(0) \le 0$.

Hui Ma (*Tsinghua University*) .... January 26th (Sat), 15:30–16:15
Lagrangian submanifolds and variational problems

This talk is based on a joint work with Professor Yoshihiro Ohnita (OCU). I will firstly give a general introduction on volume variational problem under Hamiltonian deformations, for instance, Hamiltonian minimality, Hamiltonian stability and Oh's conjecture, etc. Secondly, I will introduce an interesting link between Lagrangian Geometry in the complex hyperquadrics with Hypersurface Geometry in the unit sphere. From this viewpoint we provide a classification theorem of compact homogeneous Lagrangian submanifolds in complex hyperquadrics by using the moment map technique. Moreover we determine the Hamiltonian stability of compact minimal Lagrangian submanifolds embedded in complex hyperquadrics which are obtained as Gauss images of isoparametric hypersurfaces in spheres with $g(=1,2,3,6)$ distinct principal curvatures.

Xiang Ma (*Peking University*) .... January 28th (Mon), 16:30–17:15
Polar Transforms for Surfaces in 4-dim Lorentzian Conformal Geometry

We study spacelike surfaces in 4-dimensional Lorentzian space forms from the viewpoint of Lorentzian conformal geometry. For such a surface we define the so-called left/right polar surfaces which are again conformal maps. It is shown that these transforms preserve Willmore surfaces and isothermic surfaces, respectively. These notions are generalized from Moebius geometry and we establish a series of results which might be compared with before: the duality theorem for Willmore surfaces, the classification of Willmore 2-spheres, and the commutative theorem involving the polar transforms and other known transforms for isothermic surfaces.

Yuichi Nohara (*Tohoku University*) .... January 28th (Mon), 10:30–11:15
Toric degeneration of flag manifolds and the Gelfand-Cetlin system

The Gelfand-Cetlin polytope is related to a complex flag manifold $X = U(n)/T$ in three different theories: the Gelfand-Cetlin system, a completely integrable system on X; the Gelfand-Cetlin basis, a basis of an irreducible representation of $U(n)$; and a toric degeneration of X. It is proved by Kogan-Miller that the Gelfand-Cetlin basis can be deformed into a monomial basis on the the Gelfand-Cetlin toric variety under the degeneration. In this talk, we discuss a relation between the Gelfand-Cetlin system and the toric degeneration.

Kentaro Saji (*Hokkaido University*) .... January 27th (Sun), 14:30–15:15
Legendrian dualities for "flat" surfaces in Lorentz-Minkowski pseudo-spheres II

The "mandala" of the Legendrian dulaities between surfaces in pseudo-spheres in the Lorentz-Minkowski space was introduced by the previous talk. We study properities and relations of these surfaces from the view point of singularity theory. Let $(R^4_1, \langle\ ,\ \rangle)$ be the Lorentz-Minkowski 4-space with the inner product of signature $(-,+,+,+)$. Let $a_0(t),\dots,a_3(t)$ be a pseudo-orthonormal frame field in R^4_1 such that $\langle a_0,a_0\rangle \equiv -1$ and $\langle a_1,a_1\rangle \equiv \langle a_2,a_2\rangle \equiv \langle a_3,a_3\rangle \equiv 1$. We assume that $\langle a_2',a_2'\rangle \neq 0$ and the parameter t is arc-length with respect to the curve a_2. The surfaces are constructed from these flame. To study them, we put $c_1(t) = \langle a_0',a_1\rangle$, $c_2(t) = \langle a_0',a_2\rangle$, $c_3(t) = \langle a_0',a_3\rangle$, $c_4(t) = \langle a_1',a_2\rangle$, $c_5(t) = \langle a_1',a_3\rangle$

and $c_6(t) = \langle a_2', a_3 \rangle$. Then these functions c_* satisfy the following Frenet-Serre type formulae:

$$\begin{pmatrix} a_0' \\ a_1' \\ a_2' \\ a_3' \end{pmatrix} = \begin{pmatrix} 0 & c_1 & c_2 & c_3 \\ c_1 & 0 & c_4 & c_5 \\ c_2 & -c_4 & 0 & c_6 \\ c_3 & -c_5 & -c_6 & 0 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix}.$$

Functions $c_1, \ldots, c_6$ determine the pseudo-orthonormal frame up to the Lorentzian mortion. On the other hand, useful criteria of singularities of wave fronts are given in [1–4]. Using these criteria, we write the conditions of cetrain singularities of the surfaces only by the informations of functions $c_1, \ldots, c_6$. As an application, we introduce a duality between singular points. This is a joint work with Shyuichi Izumiya.

References

[1] S. Fujimori, K. Saji, M. Umehara and K. Yamada, *Singularities of maximal surfaces*, preprint, to apperar in Math. Z.

[2] S. Izumiya and K. Saji, in preparation.

[3] S. Izumiya, K. Saji and M. Takahashi, *Horospherical flat surfaces in hyperbolic 3-space*, preprint.

[4] M. Kokubu, W. Rossman, K. Saji, M. Umehara and K. Yamada, *Singularities of flat fronts in hyperbolic* 3-*space*, Pacific J. of Math. **221** (2005), 303–351.

Hiroshi Tamaru (*Hiroshima University*) . . . . January 29th (Tue), 11:30–12:15

Parabolic subgroups of semisimple Lie groups and noncompact homogeneous Einstein manifolds

We are interested in the geometry of solvable Lie groups. Solvable Lie groups with left-invariant metrics, solvmanifolds, provide many interesting examples of Riemannian manifolds. In this talk, we concentrate solvable Lie groups related to parabolic subgroups of semisimple Lie groups. They provide new examples of Einstein solvmanifolds. They also give us examples of homogeneous minimal submanifolds in symmetric spaces of noncompact type, which have the following property: the Ricci curvature of the submanifold coincides with the restriction of the Ricci curvature of the ambient space.

Zizhou Tang (*Beijing Normal University*) . . . . January 27th (Sun), 15:30–16:15

Changping Wang (*Peking University*) . . . . January 26th (Sat), 14:30–15:15

Willmore surfaces in S^n with constant Möbius curvature

Minimal surfaces in S^n with constant curvature K have been classified locally by Calabi, Kenmotsu and Bryant, and the constant K can only take the discrete values 0, 1 and $\frac{1}{2}m(m+1)$ for some integer m. In this talk we will study the analogous problem in Möbius geometry.

Let $x : M \to \mathbb{R}^n(\mathbb{S}^n)$ be a surface in $\mathbb{R}^n$ or $\mathbb{S}^n$ without umbilical point. Then one can define a metric $g = \frac{1}{2}(|II|^2 - 2|H|^2) dx \cdot dx$ on M which is invariant under the Möbius group. The critical surfaces of

g are called Willmore surfaces, which includes minimal surfaces in $\mathbb{R}^n$ and $\mathbb{S}^n$ as special cases. Now let $x : M \to \mathbb{R}^n(\mathbb{S}^n)$ be a Willmore surface with constant Möbius curvature K (of g). We may ask the following questions: (i) Does K take only discrete values? (ii) Can we classify such Willmore surfaces?

We can only give some partial answers to these questions. We prove that if $x : M \to \mathbb{R}^3(\mathbb{S}^3)$ is a Willmore surface with constant Möbius curvature K, then either $K = 0$ and $x(M)$ is an open part of the Clliford torus in $\mathbb{S}^3$, or $K = 1$ and $x(M)$ is an open part of any minimal surface in $\mathbb{R}^3$.

A surface $x : M \to \mathbb{S}^4$ is called isotropic, if the global defined 4-form $(x_{zz} \cdot x_{zz})dz^4$ on M vanishes. This class of surfaces are invariant under the Möbius group. We can show that if $x : M \to \mathbb{R}^4(\mathbb{S}^4)$ be a isotropic Willmore surface with constant Möbius curvature K, then either $K = \frac{1}{2}$ and $x(M)$ is an open part of the Veronese surface in $\mathbb{S}^4$, or $K = 2$ and $x(M)$ is an open part of any complex curve in $\mathbb{R}^4 = \mathbb{C}^2$.

It is a joint work with Dr. Xiang Ma, cf.: Ann Glob Anal Geom (2007) 32,297-310.

Yuanlong Xin (*Fudan University*) . . . . January 26th (Sat), 9:30–10:15

On Lawson-Osserman Problem

We derives estimates of the Hessian of several smooth functions defined on Grassmannian manifold. Based on it, we can derive curvature estimates for minimal submanifolds of higher codimension in Euclidean space via Gauss map. Thus, Schoen-Simon-Yau's results and Ecker-Huisken's results for minimal hypersurfaces are generalized to higher codimension. In this way, the previous results for Lawson-Osserman problem done by Hildebrandt-Jost-Widmen and Jost-Xin could be improved.

Hongwei Xu (*Zhejiang University*) . . . . January 27th (Sun), 16:30–17:15

Morse Inequalities and Sphere Theorems for Submanifolds

In this talk, we give some new Morse inequalities, geometric inequalities and sphere theorems for submanifolds. In particular, we obtain an analogue of the Demailly's holomorphic Morse inequalities for submanifolds. We also prove an optimal sphere theorem for 3-dimensional submanifolds in a Euclidian space and partially solve Lawson-Simons' open problem on the best pinching constant for 3-dimensional submanifolds in a sphere, which is a generalization of the sphere theorem due to K.Shiohama and H.-W.Xu [Compositio Math.,107(1997)]

Takao Yamaguchi (*Tsukuba University*) . . . . January 26th (Sat), 10:30–11:15

Two-dimensional Alexandrov spaces with curvarture bounded above —from local structure to Gauss-Bonnet

A geodesic metric space is called to have curvature bounded above if every small geodesic triangle is thiner than a comparison triangle. I will discuss local structure results for two-dimensional such spaces with an application to Gauss-Bonnet formula. This is a joint work with Bruce Kleiner, Koichi Nagano and Takashi Shioya. Historically, the notion of curvature has played crucial roles in geometry, even in mathematics. By studying this kind of singular spaces, we could know the essential feature of curvature being bounded above.

Weiping Zhang (*Nankai University*) .... January 29th (Tue), 13:30–14:15

Bin Zhou (*Peking University*) .... January 27th (Sun), 9:30–10:15

Minimizing weak solutions for calabi's extremal metrics on toric manifolds

In this talk, I will discuss a Donaldson's version of modified K-energy associated to the Calabi's extremal metrics on toric manifolds and we will establish the existence of the weak solution for extremal metrics in the sense of convex functions which minimizes the modified K-energy.

Xiangyu Zhou (*Chinese Academy of Science*) .... January 26th (Sat), 11:30–12:15

Rigidity, automorphism group and group action