GLOBAL ANALYSIS ON THOM-MATHER SPACES

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My talk concerns the class of abstract stratified spaces known as Thom-Mather (TM-) spaces, which comprises e.g. simplicial complexes, algebraic varieties, orbit spaces of proper Lie group actions on manifolds, and the classical Whitney stratified spaces. Generally, these spaces may be thought of as compactifications of a manifold by "adding" lower dimensional manifolds, possibly with common properties. As a very useful reference, one may consult John Mather's article [M] and the forgoing bibliography presented in the (illuminating) introduction by Mark Goresky (to the same issue of the Bulletin AMS).

Mather's axioms lack a clear motivation - in spite of their usefulness - and I will first try to explain their background. This leads to a topological dicussion which allows to develop a good part of the theory without any use of smoothness.

The main interest, though, lies in developing differential geometry and elliptic theory on smooth TM-spaces which presupposes, however, a certain amount of differential topology; the ground work can be found in [M], [V], and [BHS]. In this talk, I will concentrate on a flow version of the Ehresmann Theorem, and on the Whitney Embedding Theorem (into $\mathbb{R}^m$) in the context of Thom-Mather spaces. I will also review some known and some new results in the global analysis of elliptic operators, in particular, the spectral and index theory of Dirac operators, on Thom-Mather spaces. Interestingly, these spaces find increasing attention in modern mathematical physics which may lead to fruitful applications and new questions for this area of research.

References

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