WORKSHOP

GEOMETRIC ANALYSIS IN GEOMETRY AND TOPOLOGY 2016

Date : December 12th – 16th, 2016
Place :
(1) Dec. 12th– 13th : Tokyo University of Science, Morito Memorial Hall
(2) Dec. 14th– 16th : Tokyo Institute of Technology, Main building H213

Invited speakers * confirmed
· Miles Simon*(Univ. of Magdeburg, Germany)
· Boris Vertman*(Univ. of Munster, Germany)
· Thomas Walpuski*(MIT, USA)
· Richard A. Wentworth*(Univ. of Maryland, USA)
· Tsuyoshi Kato*(Kyoto Univ., Japan)
· Nobuhiro Nakamura*(Osaka Medical College)

• Fernando Coda Marques (Princeton Univ., USA) : nonparticipation by circumstances

Schedule

••••••••••	10:00–11:00	11:30–12:30	14:00–15:00	15:30–16:30
Dec. 12	Kato-1	Wentworth-1	Nakamura-1	Vertman-1
Dec. 13	Wentworth-2	Simon-1	Walpuski-1	Nakamura-2
Dec. 14	Simon-2	Walpuski-2	Wentworth-3	Kato-2
Dec. 15	Vertman-2	Simon-3		
Dec. 16	Walpuski-3	Vertman-3	Kato-3	

Organizers
• Naoyuki Koike (Tokyo University of Science)
• Shu Nakamura (University of Tokyo)
• Mikio Furuta (University of Tokyo)
• Shinichiroh Matsuo (Nagoya University)
• Osamu Kobayashi (Osaka University)
• Yoshihiko Matsumoto (Osaka University)
• Rafe Mazzeo (Stanford University, Foreign adviser)
• Kazuo Akutagawa (Tokyo Institute of Technology)
• Akiko Takagi (Tokyo Institute of Technology, Secretary)

Related Conference
The First Japan-Taiwan Joint Conference on Differential Geometry & the 8th TIMS-OCAMI-WASEDA Joint International Workshop on Differential Geometry and Geometric Analysis

Waseda University, Japan, December 13th-17th 2016 (Tuesday-Saturday).

https://sites.google.com/site/jtgeometryconference/

Titles & Abstracts

- **Tuyoshi Kato (Kyoto Univ., Japan)**

Lectures 1, 2 : Higher degree of the covering monopole map

I will introduce a monopole map over universal covering spaces of compact four manifolds. In particular we can formulate higher degree of the covering monopole map when the linearized maps are isomorphic, as an element in the equivariant E theory. It induces a homomorphism between K theory of group C^* algebras.

As an application we propose an aspherical inequality on compact aspherical four manifolds. This presents a stronger version to 10/8 inequality by Furuta, in the aspherical class of four manifolds. This holds for many cases which include some complex surfaces of general type. Technically the construction of the covering monopole map requires non linear estimates in Sobolev spaces and will motivate L^p analysis on non compact manifolds.

Lecture 3 : Computation of the degrees

The higher degree is not so easy to compute in general. We will give an explicit form of the degree when the fundamental group is isomorphic to the integer. This uses Bryan's computation of degrees for finite group actions.

We will also introduce a families of the monopole maps, and compute its degree in K theory of the parameter space. This uses a new technique which is a kind of projection method. We give a new estimate by computing higher terms of the degree in some concrete cases.

This is a joint work with Kasparov, Kawaguchi and Nakamura.

- **Nobuhiro Nakamura (Osaka Medical College, Japan)**

Lecture 1 : $\mathrm{Pin}^-(2)$-monopole theory I

$\mathrm{Pin}^-(2)$-monopole equations are a twisted version of the Seiberg-Witten equations derived from complex conjugation of spin-c structures. $\mathrm{Pin}^-(2)$-monopole theory is similar to the Seiberg-Witten theory in many points, but it has different scope of applications due to its cohomological twistedness. In the first talk, we give an introductory lecture on the generality of the $\mathrm{Pin}^-(2)$-monopole theory.

Lecture 2 : $\mathrm{Pin}^-(2)$-monopole theory II

The second talk focuses on the diffeomorphism invariants defined from $\mathrm{Pin}^-(2)$-monopole equations. In similar fashion with the Seiberg-Witten invariants, cohomological and stable cohomotopical versions of $\mathrm{Pin}^-(2)$-monopole invariants are defined. We will explain their constructions, the connected sum formulae and their applications.

- Miles Simon (Univ. of Magdeburg, Germany)

Lecture 1 : Local results and Pseudolocality for the Ricci flow

In this talk we present some of the known local results which hold for the Ricci flow, the most famous thereof being the Pseudolocality Theorem of Perelman. We give examples which show why certain conditions which are assumed in the statements of the theorems cannot be relaxed. We explain in which sense each of the results we present is 'local'.

Lecture 2 : Local estimates for the Ricci flow

In this talk we prove some local estimates for solutions evolving by the Ricci flow under various assumptions. Partly joint work with Peter Topping.

Lecture 3 : A new local result for the Ricci flow

We present a new (2016) local result for the Ricci flow and explain the proof thereof. Joint work with Peter Topping.

- Boris Vertman (Univ. of Munster, Germany)

Lecture 1 : Yamabe flow and similar evolution equations

We discuss the recent resolution of the Yamabe problem on singular edge spaces for negative Yamabe class using flow methods. We present elements of the proof and explain how these apply to establish existence and regularity of solutions to the Porous Media equation in the singular edge setting. We will also talk about possible future research directions.

Lectures 2, 3 : Ricci flow on edge manifolds and Einstein metrics I, II

We discuss the recently obtained results on the short time existence of the Ricci flow in the singular setting. Corresponding analysis in the setting of Kaehler manifolds has been used decisively in the recent resolution of the Calabi-Yau conjecture on Fano manifolds. We discuss further research directions toward proving existence of Einstein metrics with edge singularities on Riemannian manifolds.

- **Thomas Walpuski (MIT, USA)**

Lecture 1 : Singular Hermitian Yang-Mills metrics over asymptotically cylindrical Kähler manifolds

The famous Donaldson-Uhlenbeck-Yau Theorem states that any μ-stable holomorphic vector bundle on a compact Kähler manifold admits a Hermitian Yang-Mills (HYM) metric. Bando and Siu extended this theorem to reflexive sheaves. It is an interesting question to ask: under which hypothesis does a holomorphic vector bundle/reflexive sheaf over a complete non-compact Kähler manifolds admit a HYM metric? In this talk I will present joint work with A. Jacob answering this question when the Kähler manifold under consideration is asymptotically cylindrical. Understanding the behavior of a HYM near the singular set of a general reflexive sheaf is an intricate problem. A geometric desingularization scheme for reflexive sheaf invented by Rossi (and clarified by Riemenschneider) suggest a conjecture about tangent cones to singular HYM metrics. I will explain the simplest version of this conjecture and, time permitting, discuss joint work in progress on this conjecture with A. Jacob and H. Sa Earp.

Lecture 2 : $G2$-instantons over twisted connected sums

Donaldson and Thomas 's visonary article " Gauge Theory in Higher Dimensions " initiated a program to study gauge theory in the context of special holonomy and, in particular, on $G2$-manifolds. I will explain how the quest for a higher dimensional version of Chern-Simons theory naturally leads to $G2$-geometry. In joint work with H. Sa Earp, we introduced a method to construct $G2$-instantons over compact $G2$-manifolds arising as the twisted connected sum (TCS) of a matching pair of building blocks. After reviewing the TCS, I will discuss our main result and explain how to interpret it in terms of certain Lagrangian subspaces of a moduli space of stable bundles on a $K3$ surface. Finally, I will talk about how to use our construction to produce a rather concrete example of a $G2$-instanton over twisted connected sum discovered by Crowley and Nordstrom. Time permitting, I will also discuss a ideas for implementing Mukai duality for TCS $G2$-manifolds (currently being investigated in joint work with A. Kovalev).

Lecture 3 : Generalized Seiberg-Witten equations arising in gauge theory on $G2$-manifolds

It is tempting to speculate whether and how gauge theory on $G2$-manifolds can give rise to enumerative invariants. The main source of difficulty with this idea is the failure of compactness for the Yang-Mills equation in higher dimensions. The extend of this failure has been studied, among others, by Price, Uhlenbeck, Nakajima and Tian. Based on their work, Donaldson and Segal conjectured that a naive count of $G2$-instantons cannot yield an invariant because $G2$-instantons can be born/die " spontaneously ". I will discuss this death/birth phenomenon and its relation with Fueter sections of moduli spaces of bundles of ASD instanton moduli spaces. In joint work with A. Haydys, we discovered a class of generalized Seiberg-Witten equations (arising from the ADHM construction) which exhibit a similar non-compactness phenomenon. I will introduce these equations, present a concrete compactness theorem for a subclass of these equations and speculate which role these generalized Seiberg-Witten equations might play for enumerative invariants for $G2$-manifolds.

- **Richard A. Wentworth (Univ. of Maryland, USA)**

Lectures 1–3 : Hermitian-Yang-Mills connections: flows and compactification of moduli spaces