

## 第9回代数・解析・幾何学セミナー

表記のセミナーを下記の要領で開催致しますので、ご案内申し上げます。

**日時:** 2014年2月17日(月)～20日(木)

**場所:** 鹿児島大学理学部 1号館 101講義室

**世話人:** 木村 俊一 (広島大理) kimura@math.sci.hiroshima-u.ac.jp

竹内 潔 (筑波大数学) takechan@math.tsukuba.ac.jp

松村 慎一 (鹿児島大理) shinichi@sci.kagoshima-u.ac.jp

與倉 昭治 (鹿児島大理) yokura@sci.kagoshima-u.ac.jp

なお、本セミナーは、下記の支援を受けています。

基盤研究(B)/挑戦的萌芽研究 課題番号 25287007/10284150 (代表者:木村 俊一, 広島大理)

基盤研究(C) 課題番号 25400104 (代表者:竹内 潔, 筑波大数学)

若手研究(B) 課題番号 25800051 (代表者:松村 慎一, 鹿児島大理)

基盤研究(C) 課題番号 24540085 (代表者:與倉 昭治, 鹿児島大理)

鹿児島大学 学長裁量経費(若手研究者助成金) (代表者:松村 慎一, 鹿児島大理)

鹿児島大学大学院 理工学研究科 数理情報科学専攻

### ● February 17th (Monday)

**10:00～10:50:** Yoshishige Haraoka (Kumamoto University)

“Rigid holonomic systems”

**11:00～11:50:** Tomoyuki Shirai (Kyusyu University)

“ $\alpha$ -行列式とその周辺の話”

**11:50～13:30:** LUNCH

**13:30～14:20:** Toshitake Kohno (University of Tokyo)

“Conformal block bundles and Gauss-Manin connections”

**14:30～15:20:** Takayuki Hibi (Osaka University)

“ホロノミック勾配降下法入門”

**15:40～16:30:** Hiraku Kawanoue (RIMS)

“特異点解消の最近”

**16:40～17:30:** Toshiki Mabuchi (Osaka University)

“Existence problem of constant scalar curvature Kähler metrics”

### ● February 18th (Tuesday)

**10:00～10:50:** Takayuki Moriyama (Mie University)

“Deformations of special Legendrian submanifolds in Sasaki-Einstein manifolds”

**11:00～11:50:** Akira Fujiki (RIMS)

“Anti-self-dual hermitian structures on compact complex surfaces”

**11:50～13:30:** LUNCH

**13:30～14:20:** Joe Kamimoto (Kyusyu University)

“Newton polyhedra and oscillatory integrals”

14:30~15:20: Masashi Ishida (Osaka University)

“Diameter bounds under geometric flows”

15:40~16:30: Takao Akahori (Hyogo University)

“On the partially integrable almost CR structure”

16:40~17:30: Kimio Miyajima (Kagoshima University)

“孤立特異点の変形へのCRアプローチ”

18:30~ DINNER PARTY

● February 19th (Wednesday)

11:00~11:50: Hisashi Kasuya (Tokyo Institute of Technology)

“Central theorems for cohomologies of certain solvable groups”

11:50~13:30: LUNCH

13:30~14:20: Shigeki Akiyama (University of Tsukuba)

“Coincidence conditions in substitutive tiling dynamics”

14:30~15:20: Kazumasa Inaba (Tohoku University)

“On fibered links of singularities of polar weighted homogeneous mixed polynomials”

15:40~16:30: Jun-ichi Inoguchi (Yamagata University)

“曲面の可積分幾何”

16:40~17:30: Noriaki Kawanaka (Kwansei Gakuin University)

“佐藤のゲームとその仲間たち— 完全可解ゲームのはなし”

● February 20th (Thursday)

9:30~10:20: Koichi Kaizuka (University of Tsukuba)

“A characterization of the  $L^2$ -range of the Poisson transform on symmetric spaces of noncompact type”

10:30~11:20: Kazuhisa Shimakawa (Okayama University)

“On model structure of the category of diffeological spaces”

11:30~12:20: Nariya Kawazumi (University of Tokyo)

“The Goldman-Turaev Lie bialgebra and the mapping class group”

(\*) 2月18日(火)に懇親会を予定しています。(講演者以外で)出席頂ける方は、人数把握のため、2月11日(火)までに松村までメールでお知らせ下さい。また、旅費の支援が若干名できますので、希望者は松村までお知らせ下さい。人数の都合上ご希望に添えない場合もございますが、ご了承下さい。

# Abstracts of Talks

- Takao Akahori (Hyogo University)

TITLE: On the partially integrable almost CR structure

ABSTRACT: See attachment.

- Shigeki Akiyama (University of Tsukuba)

TITLE: Coincidence conditions in substitutive tiling dynamics

ABSTRACT: Substitutive tiling dynamical system is a simplest model of self-inducing structures appear in many phenomena. In this talk, I try to give an overview around them and explain the notion of coincidence, and show some recent developments if time allows.

- Masashi Ishida (Osaka University)

TITLE: Diameter bounds under geometric flows

ABSTRACT: An interesting problem on geometric flow is whether the diameters of manifolds under the evolving metrics stay bounded. Perelman proved that the diameter stays bounded for the Kahler-Ricci flow on Fano manifolds. For a general Ricci flow, in 2005, Peter Topping proved an upper bound of the diameter by applying the monotonicity of Perelman's  $W$ -entropy functional. In 2013, Qi S. Zhang refined the result of Topping and also proved a lower bound of the diameter under the Ricci flow. In this talk, I would like to discuss the problem for geometric flows including the Ricci flow as a special case.

- Kazumasa Inaba (Tohoku University)

TITLE: On fibered links of singularities of polar weighted homogeneous mixed polynomials

ABSTRACT: A mixed polynomial is a polynomial of complex variables and their conjugates. An isolated singularity of a mixed polynomial gives a fibered link. In this talk, we study fibered links which determined by polar weighted homogeneous mixed polynomials and explain the existence of mixed polynomials whose Milnor fibers cannot be obtained from a disk by plumbings of Hopf bands.

- Jun-ichi Inoguchi (Yamagata University)

TITLE: 曲面の可積分幾何

ABSTRACT: 無限可積分系とよばれている非線型偏微分方程式の多くが、微分幾何に密接に関わることが知られている。無限可積分系を構造方程式にもつ曲線や曲面の研究は「可積分幾何」とよばれるようになった。本講演では、現在、可積分幾何で関心をもたれている研究対象の中から、「3次元幾何 (Thurston 幾何) における極小曲面の構成」について解説する (時間が許せば曲線の差分幾何にも触れたい)。

- Koichi Kaizuka (University of Tsukuba)

TITLE: A characterization of the  $L^2$ -range of the Poisson transform on symmetric spaces of noncompact type

ABSTRACT: Characterizations of the joint eigenspaces of invariant differential operators in terms of the Poisson transform have been one of the central problems in harmonic analysis on symmetric spaces. From the point of view of spectral theory, Strichartz (J. Funct. Anal. (1989)) formulated a conjecture concerning a certain image characterization of the Poisson transform of the  $L^2$ -space on the boundary on symmetric spaces of noncompact type. In this talk, we employ techniques in scattering theory to present a positive answer to the Strichartz conjecture. We also discuss an image characterization of the Poisson transform for some singular cases.

- Hisashi Kasuya (Tokyo Institute of Technology)

TITLE: Central theorems for cohomologies of certain solvable groups

ABSTRACT: We show that the group cohomology of torsion-free virtually polycyclic groups and the continuous cohomology of simply connected solvable Lie groups can be computed by the rational cohomology of algebraic groups. The main theorems which are shown by elementary proofs imply many non-trivial facts on cohomology of solvable groups. In particular we have algebraic analogies with the theories of the de Rham cohomology of solvmanifolds and infra-solvmanifolds given by Nomizu, Lambe-Priddy, Mostow, Raghunathan, Baues and the Author.

- Joe Kamimoto (Kyusyu University)

TITLE: Newton polyhedra and oscillatory integrals

ABSTRACT: In the 1970s, A. N. Varchenko precisely investigated the leading term of the asymptotic expansion of an oscillatory integral with real analytic phase by using the geometry of the Newton polyhedron of the phase. Since his study, the importance of the resolution of singularities by means of Newton polyhedra has been strongly recognized. The purpose of this talk is to consider studies around this theme and to explain some joint work with Toshihiro Nose.

- Nariya Kawazumi (University of Tokyo)

TITLE: The Goldman-Turaev Lie bialgebra and the mapping class group

ABSTRACT: Let  $S$  be a compact connected oriented surface with non-empty boundary. We provide some descriptions of a completion of the Goldman Lie algebra of the surface  $S$ . From these descriptions we obtain a natural embedding of the (largest) Torelli group of  $S$  (in the sense of Putman) into the completed Goldman Lie algebra. If  $S$  has a connected boundary, the graded quotients of the embedding with respect to a filtration are exactly the Johnson homomorphisms of all degrees.

The Turaev cobracket induces a geometric constraint on the Johnson image. In particular, the Morita traces are extracted from the Turaev cobracket. If time permits, we also discuss a geometric interpretation of the Enomoto-Satoh traces. This talk is based on a joint work with Yusuke Kuno (Tsuda College).

- Noriaki Kawanaka (Kwansei Gakuin University)

TITLE: 佐藤のゲームとその仲間たち—完全可解ゲームのはなし

ABSTRACT: 佐藤のゲーム (Sato-Welter game) の研究は C.P. Welter (1954), 佐藤幹夫 (1968), J.H. Conway (1976) による基本定理 (先手と後手のどちらに必勝手順があるかを判定する代数的条件) の証明が発端であったが, その後長い間, 進展がなく孤立した特異な事例のように見られてきた. 比較的最近になって Kac-Moody Weyl 群の minuscule 元 (D. Peterson が導入した概念) に対応する佐藤のゲームの一般化が発見され, それらに対する基本定理の統一的証明も得られた. さらに「ソリトン講義録」(佐藤幹夫, 数理解析研) に短い言及があるのみであった skew な Young 図形における佐藤のゲームについても, 最近になって進展があり「佐藤のゲームの仲間たち」の全体像が徐々に見え始めてきた. これらの結果の概要を報告する.

- Hiraku Kawanoue (RIMS)

TITLE: 特異点解消の最近

ABSTRACT: 特異点解消の問題は代数幾何学における重要な問題の一つであり, 標数零では解決しているが正標数任意次元では依然未解決である. この主題について現時点で判明している範囲を明らかにし, 証明の概略を紹介する. また未解決部分が何故難しいかを説明し, これを攻略すべく近年提唱された幾つかのアプローチについても簡単に言及する予定である.

- Toshitake Kohno (University of Tokyo)

TITLE: Conformal block bundles and Gauss-Manin connections

ABSTRACT: We describe the flat connections for the conformal block bundles on a Riemann sphere as Gauss-Manin connections. We construct a period map from the homology of local systems on certain configuration spaces to the dual of the space of conformal blocks. It turns out that this period map is surjective. We show that the kernel of the period map coincides with the one for the natural map from the homology to the homology with locally finite chains.

- Kazuhisa Shimakawa (Okayama University)

TITLE: On model structure of the category of diffeological spaces

ABSTRACT: The notion of diffeological space is a generalization of the notion of smooth manifold. The category of diffeological spaces DIFF has an advantage over the category of topological spaces TOP in that it is cartesian closed without any modification. Moreover, we can develop the de Rham calculus within DIFF. In this talk, we mainly focus on homotopy theoretic aspects of the category of diffeological spaces: In particular, it will be shown that DIFF has a cofibrantly generated model structure which is Quillen adjoint to the Quillen model structure of TOP.

- Tomoyuki Shirai (Kyusyu University)

TITLE:  $\alpha$ -行列式とその周辺の話題

ABSTRACT:  $n$ 次正方行列  $A = (a_{ij})_{i,j=1}^n$  に対して,  $\alpha$ -行列式 ( $\alpha \in \mathbb{R}$ ) は

$$\det_{\alpha} A = \sum_{\sigma \in \mathcal{S}_n} \alpha^{d(\sigma)} \prod_{i=1}^n a_{i\sigma(i)}$$

と定義される. ここで,  $d(\sigma)$  は置換  $\sigma$  を互換の積で表すために必要な最小の互換の個数である. 特に,  $\alpha = -1$  の場合は行列式,  $\alpha = 1$  の場合はパーマネント,  $\alpha = 0$  の場合は行列の対角線の積に等しい. 非負定値行列  $A \succeq O$  に対して, 不等式

$$\text{per } A \geq \prod_{i=1}^n a_{ii} \geq \det A \geq 0$$

が知られており (Lieb の不等式, Fischer-Hadamard の不等式), 特に  $A \succeq O$  ならば  $\alpha = \pm 1, 0$  の場合, 任意の  $A \succeq O$  に対して  $\det_{\alpha} A \geq 0$  である. [1]において, 集合

$$\text{Pos}(\mathbb{R}) = \{ \alpha \in \mathbb{R} ; \det_{\alpha} A \geq 0 \text{ for every real symmetric } A \succeq O \}$$

について考察したが (エルミート行列に対しても同様に  $\text{Pos}(\mathbb{C})$  が定義される),  $\det_{\alpha} A$  の正值性の問題はある確率場の存在 (存在する場合は  $\alpha$ -行列式点過程と呼ぶ) と関係があり,

$$\text{Pos}(\mathbb{R}) \supset \left\{ -\frac{1}{n} ; n \in \mathbb{N} \right\} \cup \{0\} \cup \left\{ \frac{2}{n} ; n \in \mathbb{N} \right\}$$

を示した. 本講演では, この問題の確率論からの動機と関連結果を概観する.

キーワード:  $\alpha$ -行列式の定義と性質,  $\alpha$ -行列式の正值性, 点過程の相関関数としての  $\alpha$ -行列式, Wishart 行列と  $\alpha$ -行列式, マルコフ過程のグリーン核に付随する  $\alpha$ -行列式点過程のループ測度による表現.

[1] Y. Takahashi and T. Shirai, Random point fields associated with certain Fredholm determinants (I): fermion, Poisson and boson point processes, J. Funct. Anal. **205** (2003), 414–463.

[2] T. Shirai, Remarks on the positivity of  $\alpha$ -determinants, Kyushu J. Math. **61** (2007), 169–189.

[3] T. Osogami, T. Shirai and H. Waki, Remarks on positivity of  $\alpha$ -determinants via SDP relaxation, J. Math-for-Industry **5** (2013A-1), 1–10.

- Yoshishige Haraoka (Kumamoto University)

TITLE: Rigid holonomic systems

ABSTRACT: In the book “Rigid Local Systems”, N. M. Katz gave the notions of rigidity and the middle convolution, which brought a big progress to the theory of Fuchsian ordinary differential equations. We find that these notions are also useful for the study of linear holonomic systems in several variables.

Let  $\mathcal{M}$  be a holonomic system on  $\mathbb{P}^n$  with singular locus  $S$ . We can define a local monodromy along each irreducible component of  $S$ . The monodromy representation of  $\mathcal{M}$  is called rigid if it is determined by the local monodromies uniquely up to isomorphisms. Since the monodromy is a representation of the fundamental group  $\pi_1(\mathbb{P}^n \setminus S)$ , the topology of  $S$  affects the rigidity. We can also define the middle convolution for holonomic systems. These notions will bring a new development. It is also remarkable that many branches of mathematics — topology, algebraic geometry, representation theory and so on — are related.

- Takayuki Hibi (Osaka University)

TITLE: ホロノミック勾配降下法入門

ABSTRACT: ホロノミック勾配降下法は、ホロノミックな分布族の最尤推定量の計算のための、従来の手法の限界を遥かに越える、斬新な汎用的方法であり、JST CREST 日比チームの最も顕著な成果の一つである。そのアルゴリズムは  $D$  加群のグレブナー基底の理論を駆使する、代数学と数値計算の融合アルゴリズムであり、フィッシャー・ビンガム分布など、空間統計学で重要なパラメータ次元の高い分布についても適用可能である。講演では、 $D$  加群、グレブナー基底、統計学の知識を仮定せず、最尤推定の具体例を使い、ホロノミック勾配降下法を紹介する。

- Akira Fujiki (RIMS)

TITLE: Anti-self-dual hermitian structures on compact complex surfaces

ABSTRACT: The notion of anti-self-dual Riemannian metrics on an oriented smooth 4-manifold  $M$  was first introduced in mathematics by Atiyah-Hitchin-Singer in 1978. When  $M$  is a complex surface  $S$ , we may also speak of anti-self-dual hermitian metrics. We are interested in the case when  $M$  is compact. When  $S$  is Kähler, Calabi-Yau Kähler metrics on K3 surfaces are one of the most typical examples of asd-hermitian metrics and many other examples have been found since then. On the other hand, on non-Kähler surfaces except for locally flat case only known examples are those constructed by LeBrun ('91) and by Fujiki-Pontecorvo ('10). In this talk I would like to discuss the problem of existence and uniqueness of these metrics or more generally of their moduli problem (up to conformal equivalence) with emphasis on the non-Kähler case.

- Toshiki Mabuchi (Osaka University)

TITLE: Existence problem of constant scalar curvature Kähler metrics

ABSTRACT: Tian and Chen-Donaldson-Sun gave a very positive answer to the existence problem of Kähler-Einstein by solving affirmatively the Donaldson-Tian-Yau Conjecture for Fano manifolds. However, this conjecture is still open for general polarization cases. The purpose of this talk is to explain our recent work in progress by introducing a strong version of K-stability.

- Kimio Miyajima (Kagoshima University)

TITLE: 孤立特異点の変形への CR アプローチ

ABSTRACT: 複素解析空間の正規孤立特異点芽はその強擬凸境界上の CR 構造によって一意に決定され、特異点芽の変形をその境界上の CR 構造の変形によって扱うことが可能になる。本講演では、この基本的関係に基づく正規孤立特異点芽の変形の CR 表示について紹介する。

- Takayuki Moriyama (Mie University)

TITLE: Deformations of special Legendrian submanifolds in Sasaki-Einstein manifolds

ABSTRACT: A Sasaki-Einstein manifold is a Riemannian manifold  $S$  whose cone  $C(S)$  is a Calabi-Yau manifold. A submanifold  $L$  in a Sasaki-Einstein manifold  $S$  is a special Legendrian submanifold if the cone  $C(L)$  is a special Lagrangian submanifold in  $C(S)$ . In this talk, we will see that the moduli space of special Legendrian submanifolds in Sasaki-Einstein manifolds is the intersection of two deformation spaces which are smooth, and any special Legendrian submanifold admits a deformation space whose tangent space is given by harmonic 1-forms.

# On the partially integrable almost CR structure

Hyogo University  
Takao Akahori

Recently, some people in topology pay attention on contact structures. Let  $M$  be an orientable  $C^\infty$  manifold with real dimension  $2n - 1$ . Let  $D$  be the real subbundle of the real tangent bundle  $TM$ . We take a real one form  $\theta$ , satisfying  $\theta|_D = 0$  and for any point  $p$  of  $M$ ,  $\theta_p \neq 0$ . The pair  $(M, D)$  is a contact structure iff

$$\theta \wedge \wedge^{n-1}(d\theta) \neq 0.$$

Let  $(M, D)$  be a contact structure. We assume that our contact structure admits almost CR structure. This means: a complex subbundle  $S$ , which satisfies;

$$D = S + \bar{S}, S \cap \bar{S} = 0, \dim_{\mathbf{C}} \frac{C \otimes TM}{S + \bar{S}} = 1,$$

is given, then our tripple  $(M, D, S)$  is called a partially integrable almost CR structure. We note that we don't assume  $[\Gamma(M, S), \Gamma(M, S)] \subset \Gamma(M, S)$

In the case  $n = 3$ , several results have been found by topologists. In this case, even if we assume that  $(M, D, S)$  is strongly pseudo convex, we can't expect steinfillability, and the uniqueness doesn't hold (some topologists found that there are many steinfillable, which are different in the  $C^\infty$  category. But for the higher dimensional case, the situation can be different. Now suppose that our, a partially integrable almost CR structure is strongly pseudo convex and compact with  $\dim_{\mathbf{R}} M = 2n - 1 \geq 5$ . Then even for the almost CR case, we have the deformation complex like in [ALG1].

**Theorem 0.1.**

$$\Gamma(M, \mathbf{C}) \xrightarrow{\mathcal{D}} \Gamma(M, E_S^1) \xrightarrow{\bar{\partial}_{T'}^{(1)}} \Gamma(M, E_S^2) \xrightarrow{\bar{\partial}_{T'}^{(2)}} \dots$$

For the notations, see [A1].

**Proposition 0.2.** *Let  $(M, D, S)$  be a partially integrable almost CR structure. Then, there is a canonical obstruction  $o(S)$  for the CR structure is given as an element of  $\Gamma(M, E_S^2)$  and*

$$\bar{\partial}_{T'}^{(2)} o(S) = 0.$$

**Theorem 0.3.** *If  $\|o(S)\|$  is sufficiently small, then there is a  $\phi \in \Gamma(M, E_S^1)$ , and an integrable CR structure  ${}^0T''$ , which satisfies*

$${}^0T'' = \{X' : X' = X + \phi(X), X \in S\}.$$



## References

- [A1] T. Akahori, On the partially integrable almost CR structure, preprint.
- [A2] T. Akahori, The new estimate for the subbundles  $E_j$  and its application to the deformation of the boundaries of strongly pseudo convex domains, **63**(1981),pp311-334, *Inventiones Mathematicae*.
- [A3] T. Akahori, The notion of CR Hamiltonian flows and the local embedding problem of CR structures, NovaPublisher, pp79-94 ISBN 978-1-60741-011-9
- [AGL1] T. Akahori, P. M. Garfield, and J. M. Lee, Deformation theory of five-dimensional CR structures and the Rumin complex, **50**(2002), 517-549, *Michigan Mathematical Journal*.