WORKSHOP( )

GEOMETRIC ANALYSIS IN GEOMETRY AND TOPOLOGY 2013

2013 10 15 ( ) – 17 ( )
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4 1 × 3 survey

Clara Lucia Aldana (Université de Nantes)
Ben Andrews (Australian National University)
Emmanuel Hebey (Université de Cergy-Pontoise)
Rafe Mazzeo (Stanford University)

10 15 ( )

10:00–11:00
Ben Andrews (Australian National University)
" Isoperimetric profile comparison in Ricci flow and curve shortening flow "

11:30–12:30
Rafe Mazzeo (Stanford University)
" Kähler-Einstein edge metrics I "

12:30–14:00

14:00–15:00
Emmanuel Hebey (Université de Cergy-Pontoise)
" Elliptic stability for stationary Schrödinger equations I "

15:30–16:30
Clara Lucia Aldana (Université de Nantes)
" Determinants of Laplacians and compactness of isospectral closed surfaces "

10　16　( )

10:00–11:00
　Ben Andrews (Australian National University)
　" Bounds on modulus of continuity and the fundamental gap "

11:30–12:30
　Rafe Mazzeo (Stanford University)
　" Kähler-Einstein edge metrics II "

12:30–14:00

14:00–15:00
　Emmanuel Hebey (Université de Cergy-Pontoise)
　" Elliptic stability for stationary Schrödinger equations II "

15:30–16:30
　Clara Lucia Aldana (Université de Nantes)
　" Determinants of Laplacians on open surfaces "

10　17　( )

10:00–11:00
　Ben Andrews (Australian National University)
　" Non-collapsing and the Lawson and Pinkall-Sterling conjectures "

11:30–12:30
　Rafe Mazzeo (Stanford University)
　" Kähler-Einstein edge metrics III "

12:30–14:00

14:00–15:00
　Emmanuel Hebey (Université de Cergy-Pontoise)
　" Elliptic stability for stationary Schrödinger equations III "

15:30–16:30
　Clara Lucia Aldana (Université de Nantes)
　" Isospectrality for open manifolds and compactness "

- Ben Andrews :

  Overall theme : Maximum principles involving functions of several points. In these lectures I will describe a collection of related ideas centred around the idea of applying maximum principles to functions involving several points, to obtain sharp control on the behaviour of geometric equations.

Lecture 1 : Isoperimetric profile comparison in Ricci flow and curve shortening flow

I will describe a remarkably powerful method involving comparing isoperimetric profiles, which allows easy proofs of the convergence of Ricci flow on a two dimensional sphere to constant curvature (modulo scaling) and of Grayson's theorem for the curve shortening flow of embedded closed curves in the plane. This is based on joint work with Paul Bryan.

Lecture 2 : Bounds on modulus of continuity and the fundamental gap

In this lecture I will show how a simple maximum principle can be applied to get sharp control on the modulus of continuity for solutions of heat equations. While the initial motivation for this method was to get short time regularity results for nonlinear heat equations, some very interesting consequences can be obtained also from the long-term behaviour: Sharp lower bounds on the first nontrivial eigenvalue follow in several contexts. By extending the ideas to control the 'modulus of concavity' of an eigenfunction, we also prove a sharp lower bound on the 'fundamental gap'. This is joint work with Julie Clutterbuck.

Lecture 3 : Non-collapsing and the Lawson and Pinkall-Sterling conjectures

In this lecture I will describe some ideas which led to Brendle's recent proof of the Lawson conjecture: The only embedded minimal torus in S^3 is the Clifford torus. The key estimate arises from work on mine on 'non-collapsing' in the mean curvature flow. I will describe this estimate and its proof, which is similar in spirit to the ideas discussed in the previous two lectures, and then show how Brendle modified it to give the Lawson conjecture. Haizhong Li and I also used these ideas to prove a conjecture made by Pinkall and Sterling about constant mean curvature tori in the three-sphere. If time allows I will also discuss more recent results for more general classes of Weingarten surfaces.

- Rafe Mazzeo : Kähler-Einstein edge metrics

  This series of talks will focus on the existence of Kähler-Einstein metrics with edge singularities, a key step in Donaldson's program for establishing KE metrics on Fano manifolds. This relies on an interesting blend of linear and nonlinear techniques: the linear ones are drawn from geometric microlocal analysis and the nonlinear ones involve some new ways of obtaining a priori estimates for complex Monge-Ampere equations.

- Emmanuel Hebey : Elliptic stability for stationary Schrödinger equations
  _____   We discuss compactness and elliptic stability for nonlinear elliptic Schrödinger equations in the context of closed Riemannian manifolds. We describe blow-up phenomena from various viewpoints (constructive approach, a priori analysis) and discuss the progress made in the field over the past years.

- Clara Lucia Aldana :
  _____   These lectures will focus on determinants of Laplace operators and isospectrality on surfaces with singularities.

Lecture 1 : Determinants of Laplacians and compactness of isospectral closed surfaces.
   In this first talk I will explain the isospectral problem for closed manifolds. Two compact Riemannian manifolds are called isospectral if the spectrum of the Laplace operator associated to each metric is the same, including multiplicities. It is known that isospectral metrics are not necessarily isometric. In 1988, B. Osgood, R. Phillips and P. Sarnak proved compactness of isospectral sets of isometry classes of compact surfaces in the smooth topology. I will describe in some detail this compactness result. I will introduce the definition of determinant of a differential operator and will mention why their study is interesting. Time permitting, I will mention my current work on determinants of Laplacians on angular sectors and planar polygons; this last part is in collaboration with Werner Mueller and Julie Rowlett.

Lecture 2 : Determinants of Laplacians on open surfaces.
   In this talk I will consider open surfaces with finite topology whose ends are asymptotic to either cusps or funnels. I will explain the different ways to extend the definition of the zeta regularized determinant to this case. I will mention the extremal results that can be obtained in this setting. Part of the results presented here are based on my joint work with Pierre Albin and Frederic Rochon.

Lecture 3 : Isospectrality for open manifolds and compactness.
   The concept of isospectrality needs to be reformulated for open manifolds. In this talk, I consider again surfaces with ends that are asymptotic to cusps or funnels, but may have boundaries in addition. I will define the concept of being relatively isospectral, and will explain how to prove compactness of relatively isospectral sets using conformal surgeries. The results to be presented in the talk are joint work with Pierre Albin and Frederic Rochon.

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Rafe Mazzeo (Stanford University, Foreign adviser)
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