Geometry of Moduli Space of Low Dimensional Manifolds

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List of Speakers (will include)

- Yoshinobu Kamishima (Tokyo Metropolitan University)
- Marcus Khuri (State University of New York at Stony Brook)
- Shohei Ma (Nagoya University)
- Hideki Miyachi (Osaka University)
- Kunio Obitsu (Kagoshima University)
- Kenichi Ohshika (Osaka University)
- Tetsuya Shiromizu (Kyoto University)
- Asuka Takatsu (Nagoya University)
- Gilbert Weinstein (Monash University/ University of Alabama at Birmingham)

Conformally flat Lorentzian parabolic manifold

Yoshinobu Kamishima

Abstract: A Lorentzian parabolic structure contains Lorentzian similarity structure and Fefferman-Lorentz structure. In Part I, we revisit the results of Lorentzian flat geometry. It is known that a compact complete Lorentzian similarity manifold is a Lorentzian flat space form. We shall give a new characterization on compact Lorentzian flat 4-manifolds depending on *causal parallel vector fields*. This class contains the product of S^1 with a Heisenberg nilmanifold $S^1 \times \mathcal{N}/\Delta$. In part II, we introduce Lorentzian parabolic structure. A conformally flat Lorentzian geometry $(\mathrm{O}(2n{+}2, 2), S^1{\times}S^{2n+1})$ contains a subgeometry $(\mathrm{U}(n{+}1, 1), S^1{\times}S^{2n+1})$ called *Fefferman-Lorentzian parabolic* geometry. A typical example modeled on this geometry is a Fefferman-Lorentzian manifold $C(N)$ where $C(N)$ is a principal S^1-bundle over a spherical CR-manifold N. Let Γ be a discrete subgroup of $\mathrm{U}(n+1, 1)$ acting properly discontinuously on a domain Ω of $S^1 \times S^{2n+1}$. We classify compact conformally flat Fefferman-Lorentzian parabolic manifolds Ω/Γ admitting a 1-parameter group $\mathsf{H} \leq \mathrm{Conf}(\Omega/\Gamma)$. This class contains also $S^1 \times \mathcal{N}/\Delta$. Finally we discuss the deformation space of conformally flat Lorentzian parabolic structures on $S^1 \times \mathcal{N}/\Delta$.

Quasi-local mass and the static extension problem in general relativity

Marcus Khuri

Abstract: There are several competing definitions of quasi-local mass in General Relativity. A very promising and natural candidate, proposed by R. Bartnik, seeks to localize the total or ADM mass. Fundamental to understanding Bartnik's construction is an analysis of the moduli space of solutions to the static vacuum Einstein equations. Under appropriate assumptions, this leads to an existence result for a canonical geometric boundary value problem associated with these equations. The implications for Bartnik's quasi-local mass will be discussed.

The rationality of the moduli spaces of trigonal curves

Shohei Ma

Abstract: A smooth complex projective curve is called trigonal if it admits a degree 3 morphism onto the projective line. Such curves are seen as one-step higher analogue of hyperelliptic curves. Shepherd-Barron proved that the moduli spaces of trigonal curves are birationally isomorphic to the projective spaces, when the genus is congruent to 2 modulo 4. We extend this rationality result to every genus > 4. The spirit is that moduli space of algebraic varieties would be simple when the varieties are of enough special type.

Extremal length geometry on Teichmüller space

Hideki Miyachi

Abstract: Extremal length is a basic and powerful geometric quantity for studying Riemann surfaces and Teichmueller spaces. By virtue of Kerckhoff's formula, the Teichmueller distance is nothing but the extremal length spectrum metric on Teichmueller space. Therefore, the geometry on Teichmueller space in terms of extremal length is strongly related to the geometry of the Teichmueller distance.

In this talk, I would like to talk a canonical compactification of Teichmueller space in terms of the geometry on extremal length, which is recently called the Gardiner-Masur compactification. I will develop Thurston theory on the extremal length geometry on Teichmueller space. Namely, I discuss the extremal length geometry on Teichmueller space via intersection number. I also show that the Teichmueller space is canonically realized as a hyperboloid in a certain cone. This observation is comparable with Bonahon's realization of Teichmueller space via geodesic currents.

As an application of our result, I will give an alternative approach to Earle-Ivanov-Kra-Markovic-Royden's characterization of isometries. Namely, with some few exceptions, the isometry group of Teichmueller space with respect to the Teichmueller distance is canonically isomorphic to the extended mapping class group.

Deformation spaces of Kleinian groups and group actions

Kenichi Ohshika

Abstract: In recent years, attentions of specialists in Kleinian groups are focused on studying structures of deformation spaces and actions of outer-automorphisms on these spaces or their extensions (character varieties). In this talk, I shall first sketch recent progresses in this field, including solutions of fundamental problems posed by Thurston back in 1980's, and then explain how deformation spaces of Kleinian groups look. As for actions of outer-automorphisms, I shall explain conjectural pictures of their dynamics, describing in particular curious points in the varieties called primitive stable characters.

Generalised Lichinerowicz lemma, black hole uniqueness and positive mass theorem

Tetsuya Shiromizu

Abstract:

In stationary/static spacetimes, the positive mass theorem(PMT) implies us the strong restriction on the spacetime configurations. The famous one is Lichnerowicz theorem in 1955: contractible stationary vacuum spacetime manifolds are static. Since the vacuum spacetime has the zero mass, PMT tells us that the spacetime is Minkowski spacetime. But, we are often interested in non-vacuum cases. For such cases, the spacetime may have the non-trivial mass. However, we can show that the mass vanishes for some cases and then spacetime is Minkowski spacetime. this means that the non-trivial stationary configration of matters are not permitted(no-go!). Related to recent issues on the final fate of the instability in asymptotically anti-deSitter spacetimes, I will discuss a no-go in spacetimes with negative cosmological constant.

PMT also gives us powerful tool to show the uniqueness of black hole spacetimes. This was done by Bunting and Masood-ul-Alam in 1987 for vacuum black holes. Now the main topics on fundamental problems are changed to be about black holes in higher dimensional black holes or string theory set-up. I will review the recent development on the uniqueness/classilication of higher dimensional black holes bearing the essential difference between four and higher dimensions in mind.

Isoperimetric profile of radial probability measures

Asuka Takatsu

Abstract: In this talk, I discuss the isoperimetric profile of a certain absolutely continuous radial probability measure with respect to the Lebesgue measure on the Euclidean space . This is done based on the analogue of the Gaussian measure, which is proved by using Levy's isoperimetric inequality and the Poincare limit. Levy's isoperimetric inequality provides the optimal isoperimetric profile of the uniform probability measure on the Euclidean sphere, and the Poincare limit states that the m-dimensional Gaussian measure is obtained as the weak limit of the push-forward measures of the uniform probability measure on the Euclidean sphere of appropriate radius through the orthogonal projection to the first m-coordinates as dimension diverges to infinity. By replacing the orthogonal projection with a certain map, I generalize the Poincare limit, where the limit probability measure is an absolutely continuous radial probability measure with respect to the Lebesgue measure, and I estimate the isoperimetric profile of such a limit probability measure.

Mass, area, charge and angular momentum of initial data in general relativity

Gilbert Weinstein

Abstract: I will survey a number of results on inequalities between the above quantities, including the positive mass theorem and the Riemannian Penrose in- equality, with and without charge, with and without angular momentum. This will include both established inequalities, counter-examples, and conjectured inequali- ties. Most of these results include a rigidity statement whereby a unique solution is able to saturate the inequality. Hence most of these inequalities can be phrased as characterizing the unique minimizer, or unique constrained minimizer, of a function defined on the moduli space of initial data. Initial data for the Einstein equations (or Einstein Maxwell equations), must satisfy certain constraint equations, or more generally certain local energy inequalities. In the Riemannian vacuum case for example, one assume that the scalar curvature $S_g \geq 0$. A number of physically important functions are defined on the moduli space of initial data, such as the ADM mass m, the total charge Q, the total angular momentum J, and the area of the outermost horizon A. The positive mass theorem asserts that $m \geq 0$ with equality if and only if the data is that of Minkowski space. The positive mass theorem with charge states that $m \leq |Q|$ with equality if and only if the data is that of a known static solution, the Majumdar-Papapetrou solution. The Riemannian Penrose inequality states that

$$\sqrt{m} \geq \frac{A}{16\pi}$$

with equality if and only if the data is a Schwarzschild slice. These results and others in the same vein will be surveyed.