

ASSESSED COURSEWORK 4

Mathematics Tutorial II
Nagoya University
G30 Program, Spring 2012
Deadline: July 21, 14:45

Solutions should contain detailed arguments for all statements made. The points for each problem (making a total of 25 points) is indicated in square brackets. Hand in at the start of the tutorial class on July 21.

Exercise 1. Let V be the span of the vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -3 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 3 \\ 2 \\ 4 \\ -5 \end{bmatrix}.$$

in \mathbb{R}^4 . [6p]

- (a) Find an orthonormal basis of V .
- (b) Find the matrix of the orthogonal projection

$$\text{proj}_V : \mathbb{R}^4 \rightarrow \mathbb{R}^4$$

onto V in the standard basis.

Exercise 2. Determine which of the following matrices are orthogonal. [4p]

(a) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\frac{1}{3} \begin{bmatrix} 1 & 2 & -2 \\ -2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

(d) $\frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & 1 \end{bmatrix}$

Exercise 3. Find all eigenvalues of the following matrices. [6p].

(a) $\begin{bmatrix} 1 & -4 & 10 \\ 0 & 7 & -5 \\ 0 & 10 & -8 \end{bmatrix}$

$$(b) \begin{bmatrix} -4 & -6 & 16 \\ 3 & 7 & -12 \\ 0 & 1 & 0 \end{bmatrix}$$

Exercise 4. For each matrix A below, find an invertible matrix S and a diagonal matrix D such that $S^{-1}AS = D$. [9p]

$$(a) A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$$

$$(b) A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$(c) A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$