The University of Nagoya

School of Mathematical Sciences

G30 Tutorials 2, Spring 2012

ASSESSED COURSEWORK 2

Deadline: June 14th, 14:45

Exercise 1. Evaluate or explain why the limit fails to exist:

(a) $\lim_{(x,y)\to(0,0)} f(x,y) = \frac{(x+y)^2}{x^2+y^2}$.

(b) $\lim_{(x,y)\to(0,0)} \frac{x^4 y^4}{(x^2+y^4)^3}$.

Exercise 2. Evaluate the partial derivatives $\frac{\partial F}{\partial x}$, $\frac{\partial F}{\partial y}$, and $\frac{\partial F}{\partial z}$ for the given functions F.

(a) $F(x,y,z) = \frac{x+y+z}{(1+x^2+y^2+z^2)^{3/2}}$.

(b) $F(x,y,z) = \sin(x^2 y^3 z^5)$.

Exercise 3. Compute the gradient of the following functions

(a) $f(x,y) = \frac{x-y}{x^2+y^2+1}$ at $(2,-1)$

(b) $f(x,y,z) = xy + y\cos(z) - x\sin(yz)$ at $(2,-1,\pi)$.

Exercise 4. Let f be the function defined as follows:

- $f(x,y) = \frac{xy^2 - x^2 y + 3x^3 - y^3}{x^2+y^2}$ if $(x,y) \neq (0,0)$
- $f(0,0) = 0$.

(a) Calculate $\frac{\partial f}{\partial x}$, and $\frac{\partial f}{\partial y}$ for $(x,y) \neq (0,0)$.

(b) Find $f_x(0,0)$, $f_y(0,0)$.

Exercise 5. Let $z = g(x,y)$ be a class C^2 function and let $x = e^r \cos\Theta$, $y = e^r \sin\Theta$.

(a) Use the Chain rule to find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \Theta}$. Write $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ in terms of $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \Theta}$.

(b) Use Part (a) and the Chain Rule to prove that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = e^{-2r}\left(\frac{\partial^2 z}{\partial r^2} + \frac{\partial^2 z}{\partial \Theta^2}\right)$$