ASSESSED COURSEWORK 1
Mathematics Tutorial II
Nagoya University
G30 Program, Spring 2012
Deadline: June 21, 14:45

Solutions should contain detailed arguments for all statements made. The points for each problem (making a total of 25 points) is indicated in square brackets. Hand in at the start of the tutorial class on June 21.

Exercise 1. Determine which of the following sets are subspaces of $\mathbb{R}^3$. Motivate carefully. [5p]

(a) $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; x + 2y - 17z = 0 \right\}$

(b) $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; x + 2y - 17z = 12 \right\}$

(c) $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; x + y \leq y + z \right\}$

(d) $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; xy = yz \right\}$

(e) $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; x - y = y - z \right\}$

Exercise 2. Find a basis of $\mathrm{im}(A)$ [4p] and a basis of $\ker(A)$ [4p] for the following matrices A:

(a) $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & -1 \\ 1 & 3 & 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ 2 & 4 & 1 & 2 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 3 \\ 1 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 1 & 2 & -3 & 0 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Exercise 3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the reflection in the plane given by the equation $x+2y-z=0$.

(a) Find the matrix of T in the basis $\left(\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \right)$. Hint: the first two vectors are in the plane and the third is orthogonal to it. [3p]

(b) Find the matrix of T in the standard basis. [3p]

Exercise 4. Let P_3 be the space of polynomials of degree at most 3. Show that the following functions are linear transformations and find their matrices in the basis $(1, x, x^2, x^3)$.

(a) $S : P_3 \to P_3$, $S(f(x)) = f'(x) - f(x)$. [3p]

(b) $T : P_3 \to P_3$, $T(f(x)) = f''(x) - 2f'(x) + f(x)$. [3$p$]