Trigonometry

For basic definitions in trigonometry see Stewart, Appendix A. We recall the following basic formulas:

1. $\cos^2\alpha + \sin^2\alpha = 1$
2. $\sin(\alpha + 2\pi) = \sin(\alpha)$ and $\cos(\alpha + 2\pi) = \cos(\alpha)$
3. $\sin(-\alpha) = -\sin\alpha$ and $\cos(-\alpha) = \cos\alpha$
4. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
5. $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
6. $\frac{d}{dx}(\sin x) = \cos x$ and $\frac{d}{dx}(\cos x) = -\sin x$

Problem 1 Calculate

(a) $\sin(\frac{7\pi}{6})$.

(b) $\cos(\frac{5\pi}{3})$.

(c) $\tan(-\frac{5\pi}{4})$.

(d) $\sin\alpha$, when $\cos\alpha = \frac{4}{5}$ and $0 \le \alpha \le \pi$.

(e) $\cos\alpha$, when $\sin\alpha = \frac{5}{13}$ and $\frac{\pi}{2} \le \alpha \le \frac{3\pi}{2}$.

Problem 2 Find all real numbers α such that $0 \le \alpha < 2\pi$ and

(a) $\sin^2(x) = \frac{1}{4}$

(b) $\cos 5x = -1$

(c) $\sin x = \cos x$

Problem 3 Find the derivative of the function f, when

(a) $f(x) = \sin(2x)$.

(b) $f(x) = \sin(x)\cos(x)$.

(c) $f(x) = \frac{\tan(x)}{\sin(x)}$.

(d) $f(x) = \sin^3(x) - 2\cos^2 x$

Problem 4 Calculate the following derivatives for $y = \sin x$ and for $y = \cos x$.

(a) $\frac{d^2y}{dx^2}$

(b) $\frac{d^4y}{dx^4}$

(c) $\frac{d^{100}y}{dx^{100}}$

(d) $\frac{d^{123}y}{dx^{123}}$

Problem 5 Prove the following formulas using the formulas 1 – 7 and any particular values of sine and cosine that you may need. For [a] – [d] try also to justify the formula geometrically.

(a) $\sin(\pi/2 - \alpha) = \cos(\alpha)$

(b) $\cos(\pi/2 - \alpha) = \sin(\alpha)$

(c) $\sin(\alpha + \pi) = -\sin\alpha$

(d) $\cos(\alpha + \pi) = -\cos\alpha$

(e) $\sin(2\alpha) = 2\sin\alpha\cos\alpha$

(f) $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$