Matrix Multiplication

For basic definitions in matrix algebra see Bretscher, chapter 2.

Problem 1 Calculate the matrix products

(a) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 1 & 3 \\ 1 & 0 & 2 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \\ 1 & 0 & -4 \\ 0 & -1 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 5 \\ 2 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ -2 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 5 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -2 \\ 2 & 0 & 3 \end{bmatrix}$

(e) $\begin{bmatrix} 3 & 2 \\ 1 & -2 \\ 3 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 0 \end{bmatrix}$

(f) $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

Problem 2 Solve the matrix equation $AX = B$, where

(a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \\ 1 & -4 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & -2 \\ 1 & 2 & 1 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

Problem 3 Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 2 \end{bmatrix}$. Recall that $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Solve the equation

$$AX = I_3.$$

(b) Solve the equation

$$AY = B.$$

(c) Calculate XB and compare with Y. Can you explain what you find?

Problem 4 Let A be an $n \times n$-matrix of rank n.

(a) Show that there is a unique $n \times n$-matrix B such that $AB = I_n$.

(b) Show that B has rank n. (Hint: it is enough to show that $B\vec{x} = \vec{0}$ only has the solution $\vec{x} = \vec{0}$).

(c) Show that $AB = I_n$ and $BA = I_n$. (Hint: for the second statement consider the matrix equation $BX = I_n$).