Inverse Matrices

For basic definitions on inverse matrices see Bretscher, chapter 2.

Problem 1 Determine if the matrix A is invertible. If it is find the inverse A^{-1}.

(a) $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & -2 \\ -3 & 6 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & -6 \\ -3 & -5 & 8 \end{bmatrix}$

(e) $A = \begin{bmatrix} 2 & 1 & -1 \\ 5 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$

(f) $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Problem 2 Using your results from Problem 1, solve the matrix equation $AX = B$, where

(a) $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 4 & -6 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & -6 \\ -3 & -5 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$

(e) $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & -6 \\ -3 & -5 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & -2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

(f) $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 1 \\ -2 \\ -3 \end{bmatrix}$

Problem 3 Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be any 2×2-matrix. Show that

(a) A is invertible if and only if $ad - bc \neq 0$.

(b) If A is invertible then
$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Problem 4 Let
$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Calculate

(a) A^4,

(b) $I_4 + A + A^2 + A^3$,

(c) $(I_4 - A)(I_4 + A + A^2 + A^3)$.

What is the inverse of $I_4 - A$?