

Techniques of Integration

Substitution

$$\int f(g(x))g'(x) dx = \int f(u)du, \text{ where } u = g(x)$$
$$\int_a^b f(g(x))g'(x) dx = \int_{g(a)}^{g(b)} f(u)du$$

Integration by parts

$$\int f(x)g'(x) dx = f(x)g(x) - \int f'(x)g(x) dx$$
$$\int_a^b f(x)g'(x) dx = [f(x)g(x)]_a^b - \int_a^b f'(x)g(x) dx$$

Problem 1 Evaluate the following indefinite integrals using the suggested substitutions

(a)

$$\int x^2(x^3 + 4)^7 dx, \quad u = x^3 + 4$$

(b)

$$\int (x + 3) \cos(x^2 + 6x) dx, \quad u = x^2 + 6x$$

(c)

$$\int xe^{x^2} dx, \quad u = x^2$$

(d)

$$\int \frac{2x}{x^2 + 1} dx, \quad u = x^2 + 1$$

(e)

$$\int \frac{(\ln(2x))^2}{x} dx, \quad u = \ln(2x)$$

Problem 2 Find the values of the following integrals

(a)

$$\int_1^2 (x - 2)^{12} dx$$

(b)

$$\int_0^{\frac{\pi}{3}} \sin x \cos^5 x dx$$

(c)

$$\int_0^4 \frac{1}{\sqrt{x}e^{\sqrt{x}}} dx$$

(d)

$$\int_0^{\frac{\pi}{2}} \sin x \sin(\cos x) dx$$

Problem 3 Evaluate the following indefinite integrals using integration by parts

(a)

$$\int x \cos x dx$$

(b)

$$\int \ln(x+2) dx$$

(c)

$$\int x^4 \ln x dx$$

(d)

$$\int e^x \sin x dx$$

(e)

$$\int (\ln x)^2 dx$$

Problem 4 Find the values of the following integrals

(a)

$$\int_1^2 3x\sqrt{x-1} dx$$

(b)

$$\int_0^{\pi^2} \sin \sqrt{x} dx$$

(c)

$$\int_0^2 x^3 e^{x^2} dx$$

(d)

$$\int_0^{\frac{\pi}{2}} \cos^3 x \sin^{11} x dx$$