

ASSESSED COURSEWORK 4

Mathematics Tutorial I

Nagoya University

G30 Program, Fall 2012

Deadline: January 22nd, 14:45

Solutions should contain **detailed arguments** for all statements made. Each problem gives a maximum of 5 points. Hand in at the start of the tutorial class on January 22nd.

Exercise 1. Use Riemann sums to compute $\int_a^b x dx$.

Exercise 2. Compute the area bounded by the x -axis and the parabola $y = 6 - x - x^2$.

Exercise 3. Evaluate the integral $\int_0^{3\pi/2} |\sin x| dx$.

exercise 4. Compute the determinants of the following matrices

(a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(b) $B = \begin{bmatrix} 1 & -3 & 1 \\ 2 & 1 & 4 \\ -2 & 5 & -3 \end{bmatrix}$

(c) $C = \begin{bmatrix} 1 & -9 & 2 & 0 \\ 0 & 4 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & 2 & 5 & 3 \end{bmatrix}$

(d) $D = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 4 & -2 & 3 & -9 & 11 \\ 2 & 14 & 6 & 7 & -2 \end{bmatrix}$

Exercise 5. For every positive integer n define the $n \times n$ -matrix

$$A_n = \begin{bmatrix} x & 1 & 1 & \cdots & 1 \\ 1 & x & 1 & \ddots & \vdots \\ 1 & 1 & x & \ddots & 1 \\ \vdots & \ddots & \ddots & \ddots & 1 \\ 1 & \cdots & 1 & 1 & x \end{bmatrix}$$

Find all real numbers x such that A_n is invertible.