

### ASSESSED COURSEWORK 3

Mathematics Tutorial I

Nagoya University

G30 Program, Fall 2012

Deadline: December 18, 14:45

Solutions should contain detailed arguments for all statements made. Each problem gives a maximum of 5 points. Hand in at the start of the tutorial class on December 18.

**Exercise 1.** Solve the matrix equation  $AX = B$ , where

$$(a) \quad A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 0 \\ -1 & -2 \\ 8 & 1 \end{bmatrix}.$$

$$(b) \quad A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 1 \\ 1 & -4 \end{bmatrix}.$$

**Exercise 2.** Let

$$A = \begin{bmatrix} 4 & 3 & -1 \\ 3 & 0 & 1 \\ -1 & -2 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & -1 & -3 \\ 4 & 0 & 4 \\ 2 & 2 & 6 \end{bmatrix}$$

(a) Find  $A^{-1}$ .

(b) Solve the equation  $AX = B$ .

**Exercise 3.** Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ -2 & 5 & 2 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Solve the matrix equation  $(XA + B)^{-1} = C$ .

**Exercise 4.**

1. Derive the formula  $\frac{dy}{dx} = \frac{1}{1+x^2}$  for the derivative of  $y = \tan^{-1}x$  by differentiating both sides of the equivalent equation  $\tan y = x$ .
2. Show that both functions  $f(x) = \sin^{-1}\frac{1}{\sqrt{x^2+1}}$  and  $g(x) = \tan^{-1}\left(\frac{1}{x}\right)$  have the same derivative.

**Exercise 5.** Prove that

$$\tanh^{-1}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right).$$